Towards a Practical Argumentative Reasoning with Qualitative Spatial Databases

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Keywords: Argumentative reasoning, spatial representation, automated theorem proving.

Topics: AI Foundations and Knowledge Representation

Section: Paper Track

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Abstract. Classical database management can be flawed if the Knowledge database is built within a complex Knowledge Domain. We must then deal with inconsistencies and, in general, with anomalies of several types. In this paper we study computational and cognitive problems in dealing qualitative spatial databases.

1 Introduction

Spatio-temporal representation and reasoning are topics that have attracted quite a lot of interest in AI. Since the spatial notions used by the humans are intrinsically qualitative, the reasoning about spatial entities, their properties and the relationship among them, are central aspects in several intelligent systems. But the problem is far to be solved in general. The spatial reasoning is more complex than the temporal one. The higher dimension of the things is not the unique problem. The topology is, in qualitative terms, hard to represent by formalisms with amenable calculus. The semantic of these representations offers incomplete support to our daily reasoning (the *poverty conjecture*: there is no purely qualitative, general purpose kinematics). Different ontologies have been proposed, but these are not of general purpose.

Among them, the theory called *Region Connection Calculus* (hereafter referred as RCC), developed by Randell, Cui and Cohn [3] have been extensively studied in AI [10], and in the field of Geographic Information Systems [1]. A general deficiency of the theories representing topological knowledge, is that the full theory is computationally unacceptable or they fail to meet basic desiderata for these logics [8]. For constraint satisfaction problems there are algorithms to work with the relational sublanguage, and tractable subsets of the calculus RCC-8 (a relational sublogic of RCC) have been found [10]. The intractability of the full theory is mainly due to the complexity of its models (topological spaces with separation properties [6]). We propose a practical approach (using an automated theorem prover) to investigate the verification problem of knowledge bases written in the language of RCC. In general, due to the complexity of the theories

involved, the knowledge base may be inconsistent, although the environment was *well represented* by the relational part of the database. For a proper understanding of our framework, it is important to point out the following characteristics of the problem:

- The knowledge database has not ever completed (the user will write new facts in the future). Thus the difficulties begin with the future introduction of data, and the anomalies due to the present lack of them.
- The *intensional theory* of the database is not clausal. Thus, is highly possible that Reiter's axiomatization of database theory [11] becomes inconsistent. Nevertheless, the self database represents a real spatial configuration.
- The knowledge base does not contain facts about all the relationships of the RCC language. It seems natural that only facts on primary relationships appear (we have selected for our work the relations `Connect`, `Overlaps` and `Part-of`, that one can consider as the *primary* relationships).

The above characteristics are important in order to classify the anomalies. The first one may produce inconsistencies that the user can repair, but the second one is a logical inconsistency and it is hard to solve. Thus, we have to reason with inconsistent knowledge. The last characteristic implies that the (logic-based) deduction of new knowledge must be replaced to solving methods for CSP.

Our problem is only an interesting example of the more general problem of cleaning incomplete databases in the Semantic Web: the *cleaning agent* must detect anomalies in knowledge bases written by the user (in structured text), and associated to a complex ontology. It is necessary to point out that it is not our aim to find inconsistencies in the domain knowledge. In [5] the authors show an application of an automated theorem prover (the SNARK system) to provide a declarative semantics for languages for the Semantic Web, by translating first the forms from the semantic markup languages to the first-order logic. The translation allows them to apply the theorem prover to find inconsistencies. Our problem is not exactly that. We assume that the domain knowledge (the RCC theory and eventually the composition table for the relations of figure 3) is consistent, and that it is highly possible that RCC jointly with the database becomes inconsistent. However, in one of the experiments the theorem prover found an error in the composition table for the RCC-8 of [3]. Our problem has also another interesting aspect: the data inserted have not any spatial indexing.

2   The theory of RCC

The Region Connection Calculus is a topological approach to qualitative spatial representation and reasoning where the *spatial entities* are non-empty regular sets[1] (a good introduction to the theory is [3]). The primary relation between such regions is the connection relation $C(x, y)$, which is interpreted as *the closures of* x *and* y *intersect.* The axioms of RCC are two basic axioms on C, $A_1 := \forall x[C(x, x)]$ and $A_2 := \forall x, y[C(x, y) \rightarrow C(y, x)]$, plus several axioms/definitions on the main spatial relationships (see fig. 2).

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[1] A set x of a topological space is regular if it agrees with the interior of its closure.
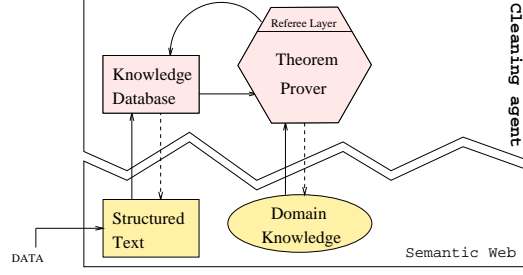
Fig. 1. Cleaning service process

Fig. 2. Axioms of RCC

The set of eight jointly exhaustive and pairwise disjoint relations shown in fig. 3 form the relational calculus RCC-8, that has been deeply studied by J. Renz and B. Nebel [10]. In that work the CSP problems associated to RCC-8 are classified in terms of (in)tractability. These problems are, in some cases, tractable, but the relational language can be too weak for some particular applications. The consistency/entailment problems in the full theory RCC have a complex behaviour. If we consider topological models, the problem is computationally unacceptable. The restriction of the problems to *nice* regions of $\mathbb{R}^2$ is also hard to compute [7].
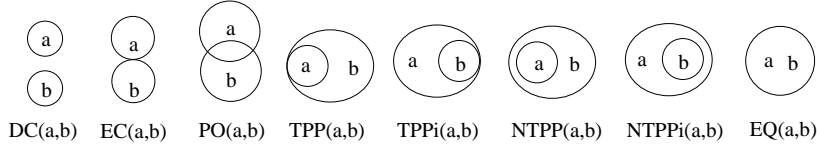

Fig. 3. The eight basic relations of RCC-8

The problem of a good representation of a model by a knowledge base arises. Concretely, we must consider three class of models: the class of all the models (according to the classical definition from first order logic), the class of the topological models, and the topological model $\mathbb{R}^n$ where the constants are interpreted as the regular sets that we want to study (*the intended model*). Formally,

Definition 1. *Let Ω be a topological space, and* X *be a finite set of constants. A structure Θ is called a* topological model *on Ω if it has the form*

$$\langle \mathcal{R}(\Omega)_{/\sim}, \mathtt{C}_\Theta, \{\mathtt{a}_\Theta \ : \mathtt{a} \in \mathtt{X}\} \rangle$$

where $\mathcal{R}(\Omega)$ is the class of regular sets, $\sim$ is the equivalence relation "the closures agree"[2], $\mathtt{C}_\Theta$ is the intended interpretation of C *and whenever* $\mathtt{a} \in \mathtt{X}$, $\mathtt{a}_\Theta \in \mathcal{R}(\Omega)_{/\sim}$.

Every structure is expanded to one in the full language of RCC, by the natural interpretation of the others relationships.

Theorem 1. *[6] If Ω is a nontrivial connected T_3-space, then the natural expansion of any topological structure on Ω to the full language is model of* RCC.

3 Towards an automated argumentative reasoning

The logic-based argument theory is a formalism to reason with inconsistent knowledge [4]. An *argument* is a pair $\langle \Pi, \varphi \rangle$ where Π is consistent and $\Pi \vdash \varphi$. The argumentative structure of K is an hierarchy of arguments which offers a method to obtain useful knowledge from K with certain properties. Also, it provides a method to evaluate the robustness of an argument via *argument trees* [2]. However, this approach can not be directly applied on big databases, since it needs, by example, to find all maximally consistent subsets of the database. The problem can be solved in the practice by adaptation the notion of *argument* to a automated theorem prover. For this paper we choose OTTER [9], a resolution-based automated theorem prover, as inference system. Since it is not our aim to describe the methodology we need to work with the theorem prover, we simply suppose that the system works in autonomous mode, a powerful feature of OTTER.

Definition 2.

1. *An* O-*argument (an argument for OTTER) is a pair $\langle \Pi, \varphi \rangle$ such that OTTER obtains a refutation from $\Pi \cup \{\neg \varphi\}$ (that we write $\Pi \vdash^0 \varphi$).*
2. *If $\langle \Pi, \phi \rangle$ is an* O-*argument, the length of $\langle \Pi, \phi \rangle$, $len(\langle \Pi, \phi \rangle)$, is the length of OTTER's refutation of $\Pi \cup \{\neg \varphi\}$*

The argumentative structure can not be directly translated, because of the consistency question. An associated automated model finder, MACE may be considered to a complete description. By example, the argument class $\mathtt{A}\exists(\mathtt{K})$ may be adapted: $\mathtt{A}^0\exists(\mathtt{K}) = \{\langle \Pi, \varphi \rangle \ : \ \Pi$ is consistent w.r.t. MACE and $\Pi \vdash^0 \varphi\}$.

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[2] The relation is necessary because of the extensionality of P given in the axiom $\mathtt{A_P}$. It produces that the mereological relation EQ agree with the equality.

4 Anomalies in complex knowldege bases

From now on, we will consider fix a topological model Θ, the spatial model on which we will work, and K a database representation of Θ, that is $\Theta \models$ K. To simplify we assume that the model satisfies the unique names axiom. Three theories describe the model: the formalization of Reiter's database theory $T_{DB}(K)$, the theory RCC(K), whose axioms are those K plus RCC, and RCC($T_{DB}(K)$). The three theories has a common language, L_K. The following is a naive ontology of anomalies in RCC-databases suggested by the experiments:

A1: The contradictions of the base due to the bad implementation of the data (some lack, other regions have not been described for making).

A2: The anomalies due to the inconsistency of the model: The theorem prover derives from the database the existence of regions which have not a name (possibly because they have not been introduced by the user yet). This anomaly can also be due to the *Skolem's noise*, produced when we work with the domain closure axioms but the domain knowledge is not clausal.

A3: Disyuntive answers (a logical deficiency).

A4: Inconsistency in Knowledge Domain.

As we remarked, the anomalies comes from several reasons: the set may be inconsistent with the Domain Knowledge due to formal inconsistencies produced by bad data, the database is not complete with respect to a basic predicate (the user will follow introducing data), etc. In fig. 4 the most common problem was shown. The system shows arguments with the skolem function of the clausal form of A_0 to questions as *gives us a region which overlaps* a.
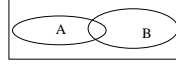

```
Database                                                  an OTTER's proof
-------------------------------------------------------------------------
all x (x=A|x=B).                                          1 [] x=x.
A!=B.                                                     5 [] x!=A|y!=B|O(x,y).
all x y (x=A&y=B|x=B&y=A|x=A&y=A|x=B&y=B->O(x,y)).       14 [] P($f1(x,y),x)|
                                                                -O(x,y).
all x y (x=A& y=B|x=B& y=A|x=A&y=A|x=B&y=B->C(x,y)).     16 [] -P(x,A)|$Ans(x).
all x y ((exists z (P(z,x)&P(z,y)))<->O(x,y)).          27 [hyper,5,1,1] O(A,B).
all x (P(x,A)->$Ans(x)).                                65 [hyper,14,27]
                                                            P($f1(A,B),A).
                                                        66[binary,65.1,16.1]
                                                            $Ans($f1(A,B)).
```

Fig. 4. A simple anomaly and an O-argument

The Skolem functions come from four axioms of RCC, A_P, A_O, A_{TPP} and A_{NTPP}, when they are clausified. It is possible to give a spatial interpretation of such

functions. Even if spatial regions are semialgebraic sets, the skolem functions can be semialgebraically defined [12]. In the practice, the spatial interpretation may be thought as a partial function. By example, the Skolem function for A_0, $f_0(x, y)$ gives the intersection region of x and y, if $O(x, y)$. This idea allow us to eliminate useless results of type (A2) extending the theory RCC with a partial axiomatization of the intersection (see the three first axioms of figure 6).

5   Consistent databases and arguments

Definition 3. *Let Θ be a topological model. The* graph *of Θ, Θ_G, is the substructure of Θ whose elements are the interpretation of the constants.*

Definition 4. *Let K be a set of formulas.*

1. *The* world *of K, $W(K)$, is the set of the interpretations in Θ of the constants in the language of L_K.*
2. *Consider an interpretation of Skolem functions of the clausal form. The* cognitive neighborhood *of K, $\Gamma(K)$, is the least substructure of the expansion of Θ to the clausal language of RCC, which contains $W(K)$.*

It seems that the consistency of an argument depends only on its cognitive neighborhood. It is true for arguments with sufficient *credibility*.

Definition 5. *An* undercut *of $\langle \Pi_1, \phi \rangle$ is an argument $\langle \Pi, \neg(\phi_1 \wedge \cdots \wedge \phi_n) \rangle$ where $\{\phi_1, \ldots, \phi_n\} \subseteq \Pi_1$. The undercut is called* local *if $\Gamma(\Pi) \subseteq \Gamma(\Pi_1)$*

Definition 6. *An argument $\langle \Pi, \alpha \rangle$ is more conservative than $\langle \Pi', \beta \rangle$ if $\Pi \subseteq \Pi'$ and $\beta \vdash^0 \alpha$.*

Definition 7. *Let T be a set of formulas, and let ϕ be a formula of the clausal language of T.*

- *A clause has* Skolem's noise *if it has occurrences of Skolem function symbols.*
- *The* credibility degree *of an argument $\langle \Pi, \phi \rangle$ is*

$$gr(\langle \Pi, \phi \rangle) = \frac{len(\langle \Pi, \phi \rangle) - |\{\eta \in Proof^0(\Pi, \phi) \; : \; \eta \; has \; Skolem's \; noise\}|}{len(\langle \Pi, \phi \rangle)}$$

The credibility degree estimates the robustness of the argument according to the use of Skolem functions in the proof, functions which becomes *ghost* regions (the credibility degree of the argument shown in fig. 4 is 4/7).

Theorem 2. *If $gr(\langle \Pi, \phi \rangle) = 1$ then $\langle \Pi, \phi \rangle \in A\exists(RCC(T_{DB}(K)))$ and $\Gamma(\Pi) \models \Pi + \phi$.*

Corollary 1. *If $gr(\langle \Pi, \phi \rangle) = gr(\langle \Pi', \phi' \rangle) = 1$ and the first argument is an undercutting of the second one, then*

- *$\Gamma(\Pi) \nsubseteq \Gamma(\Pi')$ (there is not local undercutting with credibility degree 1).*

    – If $\langle \Pi, \phi \rangle$ is a maximal conservative undercutting, then $\Gamma(\Pi') \subsetneq \Gamma(\Pi)$.

The above corollary relates undercutting arguments and spatial configurations, and it may be useful to estimates the size of *argument trees* [2].

Definition 8. *Let* K *be a knowledge database for Θ. The base* K

- *is* C-complete *if whenever* $\mathtt{a}, \mathtt{b} \in \mathtt{L_K}$, *if* $\Theta \models \mathtt{C(a,b)}$, *then* $\mathtt{C(a,b)} \in \mathtt{K}$.
- *is* extensional *for* P *if whenever* $\mathtt{a}, \mathtt{b} \in \mathtt{L_K}$

$$\mathtt{P(a,b)} \notin \mathtt{K} \quad \Longrightarrow \quad \exists \mathtt{c} \in \mathtt{L_K}\,[\mathtt{C(c,a)} \in \mathtt{K} \wedge \mathtt{C(c,b)} \notin \mathtt{K}]$$

- *is* refined *if whenever* $\mathtt{a}, \mathtt{b} \in \mathtt{L_K}$

$$\mathtt{O(a,b)} \in \mathtt{K} \quad \Longrightarrow \quad \exists \mathtt{c} \in \mathtt{L_K}\,[\{\mathtt{P(c,a)}, \mathtt{P(c,b)}\} \subseteq \mathtt{K}]$$

- *recognize frontiers if whenever* $\mathtt{a}, \mathtt{b} \in \mathtt{L_K}$ *such that* $\Theta \models \mathtt{P(a,b)}$

$$\Theta \models \mathtt{TPP(a,b)} \Longleftrightarrow \exists \mathtt{c} \in \mathtt{L_K}\,[\{\mathtt{C(c,a)}, \mathtt{C(c,b)}\} \subseteq \mathtt{K} \wedge \{\mathtt{O(c,a)}, \mathtt{O(c,b)}\} \cap \mathtt{K} = \emptyset]$$

Theorem 3. *If* K *has the above four properties, then* $\Theta_\mathtt{G} \upharpoonright_{\mathtt{W(K)}} \models \mathtt{RCC(T_{DB}(K))}$.

    An useful parameter of the graph of Θ is the *compactness level*.

Definition 9. *The* compactness level *of Θ is the least $n > 0$ such that the intersection of any set of regions of Θ_G is equal to the intersection of n of those.*

    In general, a database is not refined. Notice that when a database K is refined, the Skolem function $\mathtt{f_0}$ is interpretable within $\mathtt{W(K)}$. Thus, we can add to the database a set of axioms with basic properties of such function, and $\mathtt{f_0}$ can be *syntactly defined* in K, and simplified by compactness level, if possible. If it is nor refined, the partial definition is also useful (see fig. 7).

6 Experiments

We now report experiments with a spatial database on the relationships among three types of regions: counties, districts, and available maps on Andalucía, a Spanish autonomous region. The system works on a data base built in base of the relationships of connection (`Connect`), nonempty-intersection (common subregion, `Overlaps`), and part-of (`Part-of`). Thus there exists a lot of hidden information, a lot of knowledge with respect to other topological relations among regions, not explicit in the database, that the theorem prover might derive (and, eventually, add to the database). The graph of Θ is formed by 260 regions, approximately, for which we have a data base with 34000 facts (included first-order formalization of databases, but the number can be reduced using some features of the theorem prover). This database has been made for a human, and possibly, it contains errors. The processed database has 40242 clauses (the processing takes 6.5 seconds). It has been used OTTER 3.2 on a computer with
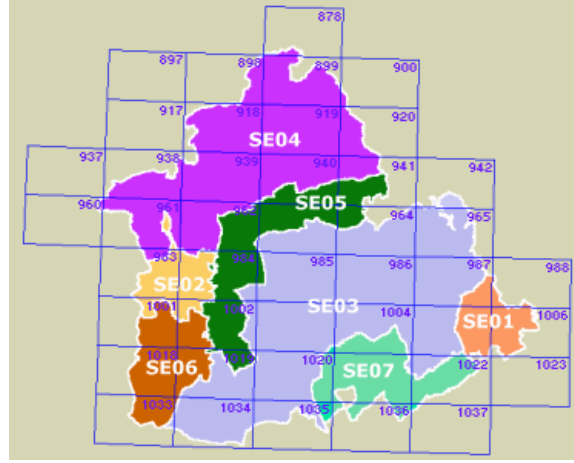
$$K = \{\langle \texttt{Connect}: \texttt{SE04}, \texttt{SE05}\rangle, \langle \texttt{Connect}: \texttt{SE04}, \texttt{Map} - 941\rangle,$$
$$\langle \texttt{Overlaps}: \texttt{Map} - 920, \texttt{SE04}\rangle, \langle \texttt{Part} - \texttt{of}: \texttt{SE04}, \texttt{SEVILLA}\rangle \cdots \}$$

Fig. 5. Partial view of the autonomous region and some facts from the database

two Pentium III (800 Mhz) processors and 256 Mb RAM. The machine runs with Red Hat Linux operating system 7.0.

The database is C-complete but it is not refined. Thus, it is highly possible that the theorem prover detects anomalies of type (A2). It recognize accidentally the frontiers, and its compactness level is 2. The compactness level can be axiomatized and incorporated to the theory if we use the (partial) spatial interpretation of the Skolem function as *partial intersection* (see figure 6). This option allows us to obtain more acceptable results, reducing the anomalies (A2) and obtaining more arguments without increase of time. (see fig. 7)

$$\texttt{Int}(x, x) = x \qquad P(x, y) \rightarrow \texttt{Int}(x, y) = x$$
$$O(x, y) \rightarrow \texttt{Int}(x, y) = \texttt{Int}(y, x)$$
$$O(y, z) \wedge O(x, \texttt{Int}(y, z)) \rightarrow \texttt{Int}(x, \texttt{Int}(y, z)) = \texttt{Int}(\texttt{Int}(x, y), z)$$
$$O(y, z) \wedge O(x, \texttt{Int}(y, z)) \rightarrow \left\{ \begin{array}{l} \texttt{Int}(x, \texttt{Int}(y, z)) = \texttt{Int}(y, z) \vee \\ \texttt{Int}(x, \texttt{Int}(y, z)) = \texttt{Int}(x, y) \vee \\ \texttt{Int}(x, \texttt{Int}(y, z)) = \texttt{Int}(x, z) \end{array} \right\}$$

Fig. 6. An axiomatization of f_0 (as Int) when the compactness level is 2

We selected the predicates Part-of, Proper-part, Externally-connect as targets of the experiments. Several results are in the figures, 7, 8 and 9. (R1) shows the first correct answer to the question, (R2) shows the results 5 seconds later, (R3) shows the first useless result and (R4) shows statistics for the first error founded.

```
P(x,Jaen) -> $Ans(x)
```

Exp.	CPU time (sec.)	generated clauses	results	(A1)	(A2)	(A3)
(R1)	54.21	175	1	0	0	0
(R1)$^+$	55,20	180	1	0	0	0
(R2)	59	671	25	102	1	0
(R2)$^+$	60,26	677	25	0	2	0
(R3)	316	19,812	232	0	5	1
(R3)$^+$	320	31,855	287	0	5	1
(R4)	54.79	570	1	0	1	0
(R4)$^+$	55.6	575	1	0	1	0

Fig. 7. Experiment on the behaviour of OTTER on a question without and with ($^+$) axiomatization of the compactness level

It is not our aim to use the theorem prover as a simple database programming language. The idea is to ask to the system complex questions which are unsolvable by constraint satisfaction algorithms or simple SQL commands. The

```
PP(x, Huelva) -> $Ans(x)
```

Exp.	CPU time (sec.)	generated clauses	results	(A1)	(A2)	(A3)	(A4)
(R1)	2395.31	195,222	1	113	0	0	0
(R2)	2400	201,797	8	113	0	0	0
(R3)	2514.46	287,088	14	117	0	1	0
(R4)	54.15	286	0	1	0	0	0

Fig. 8. Statistics for a complex question

questions are driven to obtain knowledge on spatial relationships not explicit in the database (as Proper-part or boolean combination of complex spatial relations). Some of the questions require an excessive CPU time. Surprisingly, the time cost is justified: the theorem prover *thought* all the time on the database and it found many errors of the type (A1), errors which are not acceptable. The number of useless arguments of type (A2) obtained can be significatively reduced by the spatial interpretation of f_0 function (see fig. 7).

As we remarked earlier, the theorem prover found an error in the composition table of RCC (type (A4)) working on a complex question (see fig. 9).

```
EC(x, Sevilla) -> $Ans(x):
```

CPU time (sec.)	generated clauses	results	(A1)	(A2)	(A3)	(A4)
3845	11,673,078	25	113	0	6	72

Fig. 9. Statistics of an experiment when the composition table of [3] produces errors

7 Conclusions and future work

In this paper we have focused on practical paraconsistent reasoning with qualitative spatial databases using logic-based argumentative reasoning. The problem is an example that of *cleaning* databases within complex domain knowledge, which is a promising field of applications in the Semantic Web. A spatial meaning of the arguments has shown. The next challenge is to model the robustness of an argument estimating the *number of arguments for or against* a particular sequent by topological parameters on the graph of the model. The estimation will be useful when we work with very large spatial information.

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