Chaotic series with Eco-grammar systems

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Abstract. Grammar systems are abstract models of computation invented to formalize the agents systems of Artificial Intelligence. In this work we show a way to model dynamics able to produce chaotic temporal series using Reproductive Simple Eco-grammars systems. Our system shows dynamics of population according to the logistic equation developed by May in 1973. We use an alphabet of symbols representing a population of "rabbit" agents and "carrot" resources. This work is going on the direction of approaching Artificial Intelligence, theoretical computer sciences and complex systems.

Keywords: Multiagent systems, Distributed A.I., Complex Systems, Artificial Life.

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Eco-grammar systems.

Grammar systems [4,6,8,9,12,14] are recent abstract models of computation conceived to provide of a grammatical model of distributed processes. They are agent systems where each agent is a grammar. All the agents act concurrently to perform a derivation on a common sentential form that originally modelled blackboard architectures. The agents generate collectively a language. The objective of Grammar Systems is the study of agent systems from the point of view of the pure communication protocols. This approach looks for the increment of the generative power of grammars and for the decrement of the descriptive complexity.

Eco-grammar systems [4] are a particular type of grammar systems created to model eco-systems. They are composed of an environment, a context free parallel rewriting system, or 0L system [12,15] together with a set of agents such that each agent has a set of context free rewriting rules called action rules. At each derivation step, agents act on the environmental string selecting each one a symbol and applying an action rule to that symbol, while the rest of the symbols are expanded in parallel by the environmental rules. Agents perceive the environment by means of a "sensor" function, and they have a set of internal growth rules that change an internal world. The action function selects the set of action rules taking as input the internal word.

Grammar systems and particularly Eco-grammar systems are at the cross-point of the research in Artificial Life, Complex Systems and Distributed Artificial Intelligence that is occupied in the elaboration of multiagent systems [7] following the approach of Minsky [10]. Pollack [13] said in 1989 that connectionism would introduce Artificial Intelligence in the revolution of thinking caused by physics and biology.

The relation between the research lines appears in the graph of **figure 1.** The objective of this paper is to prove that Simple Eco-grammar systems with reproductive agents are enough to show chaotic behaviour in the sequences of derived words. Simple Eco-grammars have simple agents: they do not have a sensor function (they are non sensitive) or an action function (they are fully active).

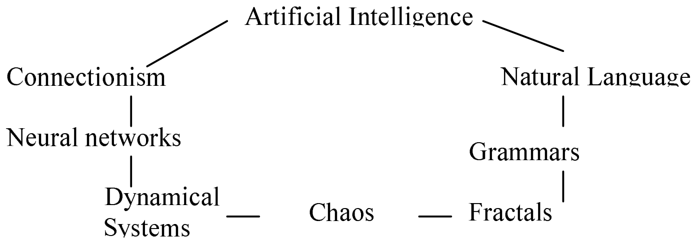

Fig. 1.

Sistemas de Eco-gramáticas simples

Eco-Grammar systems are an abstract model of computation with universal computation capability proposed as a formal frame to study systems composed of interdependent agents acting on a common environment. This model was developed by Csuhaj-Varjú, Kelemen, Kelemenova y Pa\u{u}n [4,12,14] as a formal model of ecosystems. The six basic postulates intended by the authors for the development of EG systems are:

1. An ecosystem consists of an environment and a set of agents. The internal states of the agents and the state of the environment are described by strings of symbols, called words, over certain alphabets.
2. In ecosystems there exists a universal clock that sets units of time, common to the agents and to the environment.
3. All the agents and the environment perform a parallel step of derivation at each unit of time.
4. The environmental rules are independent or the agents and independent of the state of the environment. Agents' developmental rules depend on the state of the environment, which determines a subset developmental rules that is applicable to the internal state of the agent.
5. Agents act in the environment (and possible in the internal words of other agents) according to action rules, which are rewriting rules used in chomskyan (sequential) mode. At each instant of time, each agent uses a rule selected from a set that depends on the internal state of the agent.
6. The action of the agents in the environment has greater priority than the development of the environment. Only the symbols of the environmental string that are not affected by the agents will be rewritten by the environmental rules.

Simple Eco-grammar systems are a sub-type of Eco-grammar systems composed of agents without any internal definition and without sensor and action function. A simple agent is blind; it is only a set of action rules applied in sequential mode to a symbol selected in the environmental string.

An Eco-grammar system of degree n is formally a n+2 tuple [4]:

$\Sigma = (E, R_1, \ldots\ldots, R_n, w_o)$ with $1 \leq i \leq n$ where $E = (V_E, P_E)$ is the environment and each agent R_i is identified to a set of context free rewriting. The word $w_o \in V_E^+$, is the axiom.

According to this definition, the agent will use the set of rules *Ri*, independently of the state of the environment and of the internal state. A simple eco-grammar system scheme is represented in figure 2.:

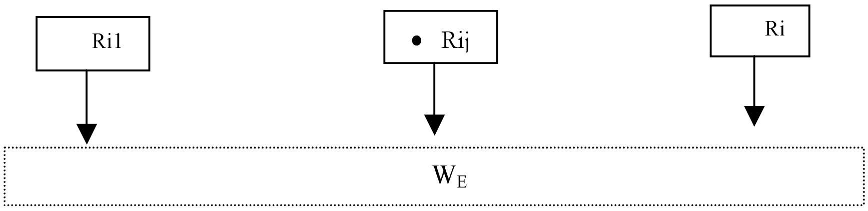

Fig. 2. Simple Eco-grammar sistem.

In our logistic system, the alphabet is composed of two symbols: $\boldsymbol{V=\{r,c\}}$. the symbol ***"r"*** represents a rabbit in the population, while ***"c"*** represents a carrot. Consider the word $\boldsymbol{w_0}$= ***cccrrrcccc*** that represents an initial population of three rabbits, and seven carrots. The agents that allow the change in the population represent active rabbits for example, $\boldsymbol{A_r=\{r\rightarrow\varepsilon, c\rightarrow r^3\}}$. The environmental rules are the set $\boldsymbol{P_E=\{r\rightarrow cc, c\rightarrow r\}}$. Symbol ε represents the empty word.

Let $\boldsymbol{\Sigma = (E, w_o)}$ be a simple eco-grammar system: formed of the environment $\boldsymbol{E=\{V,P_E\}}$ and the initial population w_0= cccrrrcccc without agents. The environmental 0L rules are the only rules, rewriting the environmental string in parallel in steps of derivation $\boldsymbol{t=0,1,2,\ldots}$

$$\mathbf{cccrrrcccc} \Rightarrow_{\Sigma} \mathbf{rrrccccccrrrr} \Rightarrow_{\Sigma} \mathbf{cccccccrrrrrrcccccccc}$$

Consider now the simple Eco-grammar system $\boldsymbol{\Sigma = (E, A_r, A_r, w_o)}$ that contains two agents $\boldsymbol{A_r}$. In a single sep of derivation, each agent selects at random one symbol in the environmental word and applies a rule. The environment expands the rest of the symbols. We underline the symbols that agents select for more clarity:

$$cc\underline{c}rr\underline{r}cccc \Rightarrow_{\Sigma} rr\underline{\ rrr\ }cccc\underline{\varepsilon}rrrr$$

Reproduction, development and death of the agents.

One of the open problems in the definition of eco-grammar systems is related to the birth, development and death of agents. This is our proposal to solve the problem:

Definition of reproduction: A simple reproductive eco-grammar system is a simple eco-grammar system where there exists at least one agent $\boldsymbol{A}$ able to generate μ-*1* copies of itself when meeting a symbol ® called reproductive symbol. Reproduction is represented as $\boldsymbol{A\rightarrow AA^{\mu-1}}$ indicating that agent $\boldsymbol{A}$ stays alive together with its descendants $\boldsymbol{A^{\mu-1}}$. This is equivalent to write $\boldsymbol{A\rightarrow A^{\mu}}$. The environment expands the rest of the symbols.

Definition of development: the new μ-1 agents generated by each agent A take positions in the environmental word after the action of the environment. They apply their rules without reproduction. This is interpreted as a delay because agents are too young to be reproduced.

Definition of death: A simple agent $\boldsymbol{A}$ dyes when in its action in the environmental string $\boldsymbol{A}$ finds a cleaning symbol ©. After using the action rule $© \rightarrow \alpha$ the agent disappears. The death of the agent is denoted $\boldsymbol{A} \rightarrow \varepsilon$.

Consequently, we define a simple reproductive agent as a 4-tuple $\boldsymbol{A=\{R,V_{®}, V_{©}, \mu\}}$ composed of a set of action rules $\boldsymbol{R}$, a set of reproductive symbols $V_{®}$, a set of cleaning symbols $V_{©}$ and the reproduction rate μ.

Logistic system: rabbits and carrots

The *logistic equation* developed by May in 1973 [16] is one of the models of population growth more studied because of the chaotic behavior that exhibit the trajectories when the system is iterated. Equation 1 corresponds to the iteration of the logistic equation from initial conditions $\boldsymbol{p(0)}$ in discrete time steps $\boldsymbol{t=0,1,2,\ldots}$ Necessarily $0 \leq \mu \leq 4$, to bound $0 \leq p(t+1) \leq 1$ ensuring that population size is N constantly over time.

$$p(t+1) = \mu \times p(t) \times (1 - p(t)) \tag{1}$$

Informal description: the system that we call logistic models a population of rabbits in a finite environment of $\boldsymbol{N}$ positions. All the positions of the environmental string not occupied by a rabbit are occupied by carrots, being the alphabet $\boldsymbol{V_E=\{r,c\}}$. Notice that agents are born and dye and hence a reproductive eco-grammar varies at each step de derivation being $\boldsymbol{\Sigma(t)}$ the grammar at step $\boldsymbol{t}$. A sequence is denoted:

$$w_0 \Rightarrow_{\Sigma(0)} w_1 \Rightarrow_{\Sigma(1)} w_2 \Rightarrow_{\Sigma(2)} \ldots \Rightarrow_{\Sigma(k-1)} w_k$$

The reproductive simple eco-grammar is $\boldsymbol{\Sigma(t) = \{E\ , A_r, \ldots^{(Nr(t))}, A_r, w_t\}}$ where $\boldsymbol{N_r(t)} = |\boldsymbol{w_t}|_r$ is the number of occurrences of symbols $\boldsymbol{r}$ in the environmental string at step $\boldsymbol{t}$, while $\boldsymbol{N_c(t)} = |\boldsymbol{w_t}|_c$ is the number of carrots. For example, if the word $\boldsymbol{w_t=crrcccr}$ has three symbols $\boldsymbol{r}$, denoted, $|\boldsymbol{w_t}|_r\boldsymbol{=3}$, the corresponding grammar will be $\boldsymbol{\Sigma(t) = \{E\ , A_r, A_r, A_r, w_t\}}$. This is equivalent to dually interpret each symbol $\boldsymbol{r}$ of the string as an agent $\boldsymbol{A_r}$. It is easy to see that the length of the word $\boldsymbol{w_t}$ is $|\boldsymbol{w_t}|\boldsymbol{=N= N_r(t)+ N_c(t)}$, constant over time $\boldsymbol{t}$.

Rabbit agents get reproduced when they find a carrot $\boldsymbol{c}$ in the interaction with the environment in a position drawn at random. Rabbit agents $\boldsymbol{A_r}$ dye when they meet a rabbit $\boldsymbol{r}$ in the string. Consequently $\boldsymbol{A_r=\{R_r,V_{®}, V_{©}\}}$ where $\boldsymbol{V_{®}=\{c\}}$ and $\boldsymbol{V_{©}=\{r\}}$.

The rewriting rules of rabbit agents are $\boldsymbol{R_r = \{r \rightarrow c,\ c \rightarrow r\}}$. The rules of the environment are $\boldsymbol{P_E = \{r \rightarrow c\ ,\ c \rightarrow c\}}$ indicating that the rabbits in the population non interacting with agents will dye proportioning substrate to a carrot, while non interacting carrots remain in the environment.

At each step of derivation, all the $N_r(t)$ agents A_r are distributed at random on the N positions of the environment. Consider the following example with w_0=***crrccccr*** and $\mu=2$. The derivation step is divided in three sub-steps:

1. ***Reproduction and death of agents***. The agents select positions and they get reproduced or dye depending on the symbol that they find in the string:

	$A_r \to \varepsilon$		$A_r \to A_r^2$			$A_r \to A_r^2$		
	$\downarrow$		$\downarrow$			$\downarrow$		
c	*r*	*r*	*c*	*c*	*c*	*c*	*r*	$=w_0$
c	***c***	*r*	***r***	*c*	*c*	***r***	*r*	

2. ***Environmental expansion***. The environment expands the rest of symbols symbols producing resources ***c***:

	$A_r \to \varepsilon$		$A_r \to A_r^2$			$A_C \to A_C^2$	
c	*c*	*r*	*r*	*c*	*c*	*r*	*r*
$\Downarrow$		$\Downarrow$		$\Downarrow$	$\Downarrow$		$\Downarrow$
c	*c*	***c***	*r*	***c***	***c***	*r*	***c***

3. ***Development of agents***. New born agents take positions on the environmental string and consume the resources created by the environmental rules. New agents are too young to be reproduced in this step, they simply apply their action rules:

	$A_r \to \varepsilon$	$\mathbf{A_r}$	A_r		$\mathbf{A_r}$	A_r		
		$\downarrow$			$\downarrow$			
c	**c**	**c**	**r**	**c**	**c**	**r**	**c**	
c	**c**	**r**	**r**	**c**	**r**	**r**	**c**	$=w_1$

Consequently, $w_0 \Rightarrow_{\Sigma(0)} w_1$ is a step of derivation. The reproductive simple eco-grammar system at step t=1 is: $\boldsymbol{\Sigma(1) = \{E , A_r, A_r, A_r, A_r, w_t\}}$

The dynamic of the system

To determine the dynamic of the system we have to determine the population of rabbits X_r and of carrots X_c at time step $t+1$ from the population at step **t**. The length of the derived word is constantly N and the number of rabbit agents is identical to the number of symbols r in the string if we start at initial conditions filling this requirements at $t=0$:

$$X_r(t+1) = \mu \times X_r(t) \times \frac{X_c(t)}{N} = \mu \times X_r(t) \times \frac{(N - X_r(t))}{N} \qquad \textbf{(2)}$$

$$X_c(t+1) = N - \mu \times X_r(t) \times \frac{X_c(t)}{N} = N - \mu \times X_r(t) \times \frac{(N - X_r(t))}{N}$$

We call $p_r(t) = p(t) = \frac{X_r(t)}{N}$ to the probability of finding a symbol r in the environment w_t.

The probability of meeting a carrot is $p_c(t+1)=1-p(t)=\frac{N-X_r(t)}{N}$.

Dividing the two members of equation, 2 by N we obtain a two dimensional systems given in equation 4. the logistic equation(1):

$$p_r(t+1)=p(t+1)=\mu\times\frac{X_r(t)}{N}\times\frac{(N-X_r(t))}{N}=\mu\times p(t)(1-p(t)) \qquad \textbf{(3)}$$
$$p_c(t+1)=1-p(t+1)=1-\mu\times p(t)(1-p(t))$$

Analysis of the stability in the equilibrium.

An equilibrium point for equation 1 is a pair $x^*=(p^*,1-p^*)$ such that $p^*=p(t)=p(t+1)=\mu p(t)(1-\ p(t))$ and $p_c^*=1-p^*=1-p(t+1)=1-p(t)$. Thus, $\mu\ p^*(1-p^*)=p^*$ obtaining two equilibrium points, corresponding to values $p_1^*=0$, and $p_2^*=1-1/\mu$. The equilibrium points of the logistic equation are $x^*_1=(0,1)$ and $x^*_2=(1-1/\mu,1/\mu)$ respectively.

The first case correspond to the stationary state $x^*_1=(0,1)$ reached when all the agents dye. The second corresponds to an equilibrium point whose stability depends on the value of μ. There are three types of equilibriums: stable, unstable and indifferent. An equilibrium point is stable when any perturbation affecting value p^* tends to disappear over time, that is the system tends to recover the equilibrium value p^*. The equilibrium is unstable if the effect of the perturbation tends to increase, an indifferent if the perturbation stays in a neighborhood of p^* in the long term behavior of the system. Notice that p^* is a stable equilibrium of the logistic equation 1 iff $x^*=(p^*,(1-p)^*)$ is a stable equilibrium of the two dimensional systems of equation 3 representing the dynamic of the reproductive simple eco-grammar system.

Equation 1 is the discrete time version of the logistic function $F_\mu(p)=\mu p(1-p)$. Starting at initial conditions $p(0)$, we obtain the iteration $p(t+1)=F_\mu(p(t))=\mu p(t)(1-p(t))$. At each time step t, the increment of a perturbation of the equilibrium p^* is a geometrical succession where the first derivative of F_μ is the growth rate, we obtain the condition for stability by the method Lyapunov coefficient λ as explained in [16]:

$$\lambda=\left|\frac{\partial F\mu(p)}{\partial p}\right|_{p^*}=\mu-2\mu\ p^* \qquad \textbf{(4)}$$

If $|\lambda|<1$ the equilibrium is stable. If $|\lambda|>1$ the equilibrium is unstable. If $|\lambda|=1$ the equilibrium is indifferent.

Analyzing value $\lambda=\mu-2.\ \mu p^*$ in the equilibrium p^*:

For $x_1^*=(0,1)$; $p_1^*=0$ and $\lambda=\mu$

For $x_2^*=(1-1/\mu,1/\mu)$; $p_2^*=1-1/\mu$ and $\lambda=2-\mu$.

From order to chaos.

In this section we analyze the equilibriums p^* for the values of $\mu \in N$ between 1 and 4. Notice that any real value μ could be considered if we consider probabilistic agents, that is agents with a distribution of probability applied to the rules with the same left hand side.

μ = 1 A slow walk to extinction.
The only equilibrium is in this case $\boldsymbol{x_1^*=x_2^*=(0,1)}$, that is stable, and means that rabbits will dye in the long term evolution of the system.
$|\lambda| = |-1| = |2-1| = 1$ and consequently the equilibrium is indifferent. In our system, agents A_r and with with them symbols r in the environment, tend slowly to disappear.

μ = 2 The way to stability.
The equilibriums are $x_1^*=(0,1)$ and $x_2^* = (1/2,1/2)$ and respectively:
$|\lambda|_1 = |-2| = 2>1$ is a unstable equilibrium,
$|\lambda|_{1/2} = |2-2| = 0<1$ is stable and reached evolutionarily.
The observation of these two results indicates that the introduction of agents A_r in the system and symbols r in the environmental string in equilibrium state x_1^* brings the system to reach the point x_2^*.

μ = 3 More or less.
The equilibriums are $x_1^*=(0,1)$ and $x_2^*=(2/3,1/3)$ and respectively:
$|\lambda|_1 = |-3| = 3>1$ is unstable,
$|\lambda|_{2/3} = |2-3| = 1$ is indifferent, non reachable evolutionarily.
Introducing agents A_r in the system and symbols r in the environment in the state x_1^* makes the system go towards state x_2^*, but the point is not reachable evolutionarily. The trajectories will approximate to the point x_2^* with smaller oscillations over time.

μ = 4 Chaos:
The equilibriums are $x_1^* = (0, 1)$ and $x_2^* = (3/4, 1/4)$ y, respectively:
$|\lambda|_1 = |-3| = 3 > 1$ is unstable
$|\lambda|_{3/4} = |2-4| = 2>1$ is also unstable.
When agents A_r and symbols r are introduced in the system in the state x_1^*or x_2^* the system does not tend to recover the equilibrium.

For the purpose of this work we are interested in the value $\mu = 4$, that shows chaotic behavior.

Is Chaotic the behavior of the system?

The more exact meted to determine the presence of chaos in a system consists in calculating the maximal Lyapunov coefficient [16] that describes the evolution of two trajectories starting at initial conditions arbitrarily near. The Lyapunov coefficient measures one of the more characteristic properties of chaotic series, the sensibility to initial conditions.
In the case of dimension 1, like the logistic equation, the exponent has an intuitive meaning. It is the exponent that regulates the distance between the points of two near trajectories.
If the exponent is negative, the two trajectories tend to converge one to another because the distance is decreasing over time. If the exponent is zero, the distance the trajectories are parallel. That is the case of periodic or quasi-periodic trajectories. If the Lyapunov exponent is positive, the two trajectories tend to separate one to another. The velocity of convergence or divergence of two trajectories is what the Lyapunov exponent tells us.

The *number* ***or*** *coeficcient of Liapunov* for the logistic equation is calculated as follows. Consider:

$$\kappa = \lim_{t\to\infty} \left(J(t) \right)^{1/t} \tag{5}$$

where:

$$J(t) = \left| \left(\frac{\partial F\mu(p)}{\partial p} \right)_{p(1)} \right| \cdot \left| \left(\frac{\partial F\mu(p)}{\partial p} \right)_{p(2)} \right| \cdots\cdots\cdots \left| \left(\frac{\partial F\mu(p)}{\partial p} \right)_{p(t)} \right| \tag{6}$$

The Liapunov coefficient is obtained calculating the neperian logarithm:

$$\ln \kappa = \lim_{t\to\infty} \frac{\ln J(t)}{t} \tag{7}$$

We calculated the Lyapunov coefficient for 400 iterations starting at $x(0)=(0.04,0.996)$ for the value $\lambda=4$. The number is $\ln\ \kappa= 0,69457843 > 0$. Consequently, two trajectories diverge exponentially, the temporal series representing the values $p(t)$ are chaotic, and hence $x(t)$ defines a chaotic trajectory.

Conclusions

In this work we develop a way to model chaotic temporal series by means of grammatical agents systems called Grammar Systems [4,6,8,9,12,14]. The aim of Grammar Systems is to model the agents systems of Artificial Intelligence [7,10] as pure protocols. An specialised variant of Grammar systems are Eco-grammar systems[4]. They are composed of agents that represent organisms in an ecosystem and an environment. Eco-grammar systems are useful to analyse the simulations of ecosystems elaborated in the field of Artificial Life[3] and to model evolutionary games [2]. This work goes on the line of Pollack[13], relating Artificial Intelligence to Formal language theory and to the field of Complex Systems and Artificial Life.

We elaborate an agents model of the logistic equation $p(t+1)=\mu p(t)(1-p(t))$ [16] using Reproductive Simple Eco-grammar systems. The definition of birth, development and death of agents is an open problem in [4] that we solve in this paper.

In this work, for the sake of simplicity, we use a factor μ of reproduction that is a natural number, but any non-negative real number could be obtained using agents with probabilistic rules and environments [1,3].

We chose the representation of the logistic equation in grammatical terms because of its simplicity and because of the complexity of its behaviour. We proved that chaotic temporal series could be obtained as the mean dynamic of populations of symbols generated by Reproductive Simple Eco-grammar systems. In a further extension of this paper we will prove that adding predators to the system, wolf agents A_w and symbols w, we get the classical equations modelling the population of predators and prays of Lotka-Volterra [16].

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Bibliografía

[1] **Alfonseca, M.; Ortega A. and Suarez A.** Celular Automatas and probabilistic L Systems in [9]

[2] **Anchorena** S. **and Cases B.;** Eco-grammars to model biological systems: adding probabilities to agents". Proceedings of the European Conference on Artificial Life, ECAL 2002. Springer. Berlin (2001)

[3] **Cases, B. and Alfonseca, M**. *Eco-grammar Systems applied to the analysis of the dinamycs of populations in artificial Worlds,* in[9]

[4] **Csuhaj-Vatjú, E., Kelemen, J., Kelemenová, A y G. Paŭn**: *Eco-Grammar Systems: A Grammatical Framework for Life-Like Interactions, in Artificial Life volumen 3 #1* (winter 1997), pp 1-38.
[5] **Csuhaj-Varjú, E.; Dassow, J.; Kelemen, J., Paŭn, G. (1994)** *Grammar Systems. A Grammatical Approach To Distribution And Cooperation*. Gordon and Breach Science Publishers (1994)
[6] **Dassow, J., Paŭn, G. & Rozenbberg, R.** Grammar Systems, in *[14]*
[7] **Erceau, J. Ferber, J**.. *"La Inteligencia Artificial Distribuída"* Mundo Científico. n'l l6 vol l 1. pp.850-858
[8] **Kemelonova, A, Freund, W.** *Grammar Systems 2000*, Silesian University, Czech Republic, 2000.
[9] **Martín Vide, C. and Mitrana V.** *Words, sequence, grammars, languages: where biology, computer science, linguistics and mathematics meet",* volume=II, Springer London (2000)
[10] **Minski, M**. the *Sociedad of the Mente. La Inteligencia Humana A la Luz of the Inteligencia Artificial.* Ediciones Galapago. Argentina 1986
[11] **Montero, M. y F.Morán,** *Biofísica: Procesos of Auto-organización in Biología*, Eudema Universidad, Madrid, 1992.
[12] **Paŭn, G. (Ed.)** *Artificial Life: Gramatical Models*. Black Sea University,
[13] **Pollack, J.B.** -Connectionism: past, present, and future- Artificial Intelligence Review nº 3, 1989
[14] **Rozenberg, G. Salomaa, A. (Eds.) (1997)** Handbook Of Formal Languages. Vol 1, 2, 3. Springer
[15] **Salomaa, A.** Developmental Models for Artificial Life: Basics of L Systems. in Paun, G. (Ed.) *Artificial Life: Gramatical Models*. Black Sea University, Bucharest, Romania, 1995.
[16] **Solé, R. & Manrubia, S.** *Orden y Caos in Sistemas Complejos.* Edicions UPC, Barcelona 1996.