COVER SHEET

Bayesian approach based on geometrical features for validation and tuning of solution in deformable models

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Abstract. A local deformable-model-based segmentation can be very helpful to extract objects from an image, especially when no prototype about the object is available. However, this technique can drive to an erroneous segmentation in noisy images, in case of the active contour is captured by noise particles. If some geometrical information (a priori knowledge) of the object is available, then it can be used to validate the results and to obtain a better segmentation through a refining stage. A Bayesian approach is proposed to evaluate the solution of segmentation performed by a deformable model. The proposed framework is validated for vessel segmentation on mammograms. For that purpose a specific geometric restriction term for vessels on mammograms is formulated. Also a likelihood function of the contour and the image is developed. Our model avoids manual initialization of the contour using a local deformable model to obtain an initial contour approximation.

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Introduction

A local deformable-model-based segmentation scheme can overcame some limitations of the traditional image processing techniques and no model template about the object is needed. The inherent continuity and smoothness of the models can compensate for gaps and other regularities in object boundaries. Unfortunately, this technique presents problems in noisy images. However, if some information of the model such us geometrical features is available, then some problems caused by the noise or edges from another objects in the image can be solved using a global deformable model.

Several segmentation-scheme based on snakes (local deformable model) have been developed for noisy images [1,2] but none of them are applicable for segmentation in images with granular noise (macro-particles) or when objects are superimposed. We present a model that is adequate for those cases where some geometrical information about the object is available.

This paper is organized as follows. In the next section a brief review of previous work on geometrical deformable models is given. The limitations of these proposed methods and some open problems are pointed out. Our new approximation to a geometrical (global) deformable model is presented in Section 3, where a Bayesian approach of the geometrical constraints is described. In section 4, two different algorithms, global minimization and fast global minimization are proposed. In Section 5 the results show that our new geometrical deformable model is helpful to segment objects with granular noise or another superimposed objects. Finally, in Section 6, conclusions are presented.

Geometrical deformable models

Deformable models are a useful tool for image segmentation. They can be classified, according to the information held by the model [3], in local deformable models and global deformable models. Local deformable models manage information to pixel level taking into account only a close pixel neighborhood. However, global deformable models can use information from any location in the image. Local deformable models are faster to converge and they do not need a template of the object to be segmented. The most popular approach to local deformable model is the "snake" by *Kass et al*[4]. Unfortunately, they are very sensitive to noise. On the other hand, global deformable models have slower convergence than local deformable models and usually they need a template of the object. However, global deformable models are more robust and less sensitive to noise than local models.

Geometrical deformable models are a particular case of global deformable models that use geometric information of the object. Geometrical models have been used for a number of applications in image segmentation. Continuous geometric models consider an object boundary as a whole and can use the a priori knowledge of object shape to constrain the segmentation problem. Wang and Gosh [5] proposed a geometrical deformable model based on the curve evolution theory in differential geometry. Burger et al [6] used a geometrical priori knowledge information for the segmentation of the aorta. Delibasis and Undrill [7] developed a geometric deformable model for anatomical object recognition. And Clarysse and Friboulet [8] proposed a 3-D deformable surface model with geometrical descriptors for the recognition of the left-ventricular surface of the heart. Shen and Davatzikos [9] used an attribute vector to characterize the geometric structure around each point of the snake and only affine-segment transformations of a standard shape are allowed during convergence process. This restriction permits a faster convergence but a close object initialization has to be carried out. In addition, some kind of objects can not be properly represented using this standard shape.

Our geometric model avoids guided initialization by using a previous local deformable model where no user interaction is required.

Description of geometric deformable model

In the segmentation task, we assume the object to be detected is represented by a contour(x) in an image(z). A polygon (or vector) representation of an object is a representation where the contour is defined by a set of nodes giving coordinates of contour points in a circular (clockwise) manner. Between each node, the contour is defined by a straight line (or some spline-curves).

The Bayesian paradigm consist of four successive stages:

1. Construction of a prior probability distribution $\pi(x)$ where x is the contour of the object. Priori knowledge is included in $\pi(x)$.
2. Combining the observed image z with the underlying contour x through a conditional probability density $f(z \mid x)$.
3. Constructing the posterior density $p(x \mid z)$ from $\pi(x)$ and $f(z \mid x)$ by Bayes Theorem giving

$$p(x \mid z) \propto \pi(x) \; f(z \mid x) \tag{1}$$

4. Base any inference about the contour x on the posterior distribution $p(x \mid z)$

Prior knowledge is usually present on the contour to be recognized. This prior knowledge is associated to features as shape, size, orientation, etc.

In Bayesian analysis, all kinds of inference are calculated from the posteriori probability $p(x \mid z)$. Finding the maximum a posteriori (MAP) estimation is the most used choice of inference. Constraints on contours are, for this approach, designed through energy-functions. The energy-function consists of internal energy and external energy, where the internal energy is related to geometric features of the contour and external energy is related the contour and the image. Assume $U(x)$ is the total energy of the contour represented by x, and given by

$$U(x; z) = U_{\text{int}}(x) + U_{ext}(x; z) \tag{2}$$

where U_{int} is the internal energy while U_{ext} is the external energy (depending on the observed image z). Then, a prior model is defined by

$$\pi(x) = \frac{1}{Z_{\text{int}}} e^{-U_{\text{int}}(x)} \tag{3}$$

where Z_{int} is a normalization constant guaranteeing the prior model to be a proper probability distribution.

Assume further that the likelihood for the observed image z given x is defined by

$$f(z \mid x) = \frac{1}{Z_{ext}} e^{-U_{ext}(x;z)} \tag{4}$$

where Z_{ext} is a normalization constant similar to Z_{int}. The posterior probability for the contour conditioned on the image is then

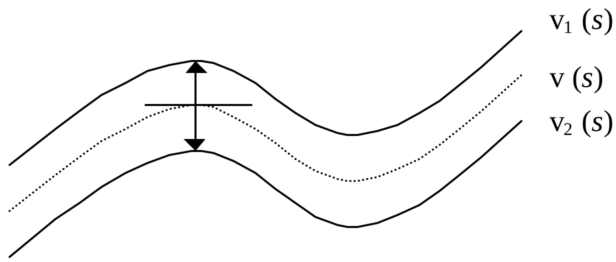


Fig. 1. Vessel geometric model

$$p\,(x\,|\,z) \propto p\,(x)\,f\,(z\,|\,x) \propto e^{-U_{\mathrm{int}}(x)-U_{ext}(x;z)} = e^{-U\,(x;z)} \qquad \textbf{(5)}$$

verifying that minimization of the energy function corresponds to maximization of the posterior distribution, that is, finding the Maximum a posteriori (MAP) solution.

This Bayesian framework is proposed in order to evaluate the contour solution through a posteriori probability (APP) according to the $p(x\,|\,z)$ function and a threshold of reliability q_p. The prior model, where the geometrical information is incorporated, is discussed in section 3.1 and the probability densities for observed image are presented in section 3.2.

Prior model and geometrical information

Let's consider that the contour x has a polygon representation $x = (p_0, p_1,.., p_{n-1})$ (contour of n points) with $p_0 = p_n$ and where p_i gives the coordinates of a point on the contour. A priori knowledge is present in the prior model through the $\boldsymbol{p}(x)$ distribution from Eq. 3. The *a priori* distribution should capture the knowledge available about x. A common assumption is that the energy-function is built up by potentials measuring local and global characteristics. To incorporate the *a priori* geometrical information in the Bayesian approach, a new potential, U_{geom}, is considered to capture the geometrical information. Then,

$$U_{\mathrm{int}}(x) = a \cdot U_1(x) + b \cdot U_2(x) + g \cdot U_{geom}(x) \qquad \textbf{(6)}$$

where $U_1(x)$ and $U_2(x)$ are the typical continuity and curvature term respectively [4]. In addition, $U_{geom}(x)$ is the geometrical restriction term, and a, b

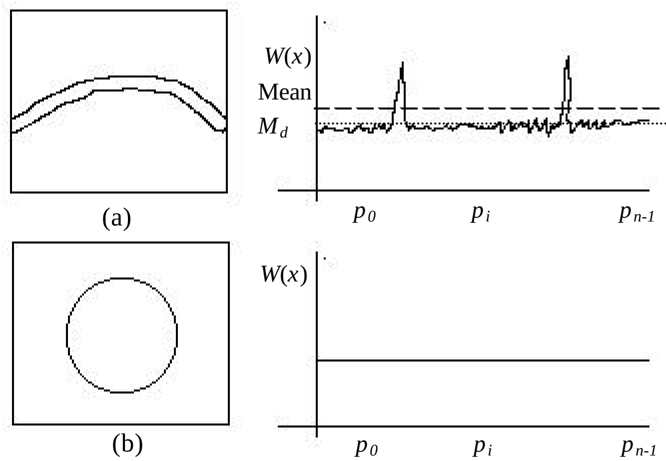


Fig. 2. Width function $W(x)$ for a vessel shape (a), and a circle shape(b)

and g parameters are regularization factors . In the following the term U_{geom} is developed.

Geometrical constraints

Geometrically, a vessel can be defined as two parallel edges having a distance between them (width of the vessel) within a range. Let's consider that $v(s)$ with $s \in [a,b]$, is the axis curve of the vessel. Then, the edges of the vessel, $v_1(s)$ and $v_2(s)$ (Figure 1), can be defined as:

$$v_1(s) = v(s) + l/2 \frac{\nabla v(s)}{\|\nabla v(s)\|}, \text{ y} \tag{7}$$

$$v_2(s) = v(s) - l/2 \frac{\nabla v(s)}{\|\nabla v(s)\|},$$

with $s \in [a,b]$, where $\nabla v(s)$ is the gradient vector of s and l is the width of the vessel. That is, the two edges and the axis have the same gradient vector in a point s.

Given a contour point p_i , we say that fp_i is the frontal point of the p_i, if fp_i is the contour point that intercept the line of gradient direction (perpendicular direction to the contour) of the point p_i. Then, we define the width of the contour at a point p_i , $W(p_i)$, as:

$$W(p_i) = distance(p_i, fp_i) \qquad \textbf{(8)}$$

that is, the distance between p_i and its frontal point. The width function of a vessel is represented in Figure 2a. Note that this function is dependent of the beginning point p_0. Also can be observed that this function is almost flat but two peaks. Because an image has limited dimensions, just a fragment of the vessel is present in it, and two additional edges are produced where the vessel crosses the borders of the image. These edges produce the two peaks in the width function of the vessel.

Most values of the width function are closer to the medium value (M_d) than to the mean value of the function because the median value is less sensitive to extreme values (two peaks in width function) than the mean value. Then we can think that a shape is more similar to a vessel shape in the way that its width function is more similar to a flat width function. However a circle shape has a flat width function (Figure 2b), then additional features such us compactness or elongation have to be taken into account. Finally, the U_{geom} potential function is defined as:

$$U_{geom}(x) = m \frac{\sum_{i=0}^{n-1} \left| W(p_i) - M_d \right|}{n \cdot s_{Max}} + l \frac{C(x)}{4p} \qquad \textbf{(9)}$$

where the first term is an estimation of the normalized deviation of the $W(p_i)$ function and s_{Max} is the maximum deviation possible of $W(p_i)$. Similarly, $C(x) = area/(perimeter)^2$ is the fractal dimension of the object and is minimal for a disk-shaped region. The parameters m and l are regularization factors between both terms, and they must verify $m + l = 1$ in order to be keep the term U_{geom} normalized.

Probability densities for image data

The probability density $f(z \mid x)$ is related to the specification of the observed image data. In our approach it is defined through an external energy, which depends on the contour and the observed image. The external energy function connects the contour to the image features were defined through potentials along the contour. Thus, a generic external energy function can be defined as:

$$U_{ext}(x; z) = -\sum_{i=0}^{n-1} h(p_i; z) \qquad \textbf{(10)}$$

where $h(p_i, z)$ is some local measure from the observed image z at location p_i (contour pixel). Usually, measure is based on grey-levels itself or some gradient value. In many cases, the complexity of the segmentation process does necessary to take into account not only the observed measurements along the contour but also the

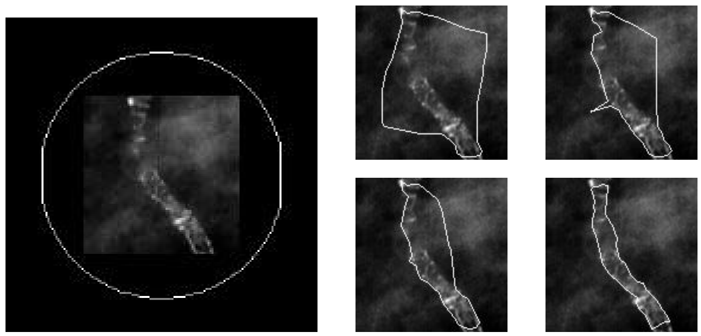


Fig. 3. Initial contour and results with the local deformable model

gray level inside and outside the contour. For example, the average gray-level inside the object is very different from the average outside the object. Such information can be very useful but it will increase the time of computation with respect to the local measures. In this case, an alternative model is to assume one distribution function f_1 for the pixels inside the object, and another distribution f_2 for the pixels outside the object, then

$$U_{ext}(x:z) = -\left| \frac{\sum_{p_j \in R_1} g(p_j)}{|R_1|} - \frac{\sum_{p_k \in R_2} g(p_k)}{|R_2|} \right| \tag{11}$$

where R_1 and R_2 are the sets of pixels inside and outside the contour, respectively, being R_1 and R_2 specified by x. Moreover, $g(p_i)$ is the gray value of the pixel p_i in the image.

Minimization

As pointed in previous section, maximizing the posterior distribution corresponds to minimizing the energy function. Optimal solutions are usually hard to find. Minimization algorithms can be based on dynamic programming [10], variational calculus [4] or iterative exploration [11], which is the most popular approach. We use this approach for our model. This method consists in dynamically move the contour towards the optimal solution. In the following the initial contour and iterative exploration of our methods are presented

Initial contour

Usually, deformable global models have longer computation time than local deformable models. A critical factor in computation time is the initial contour. Some

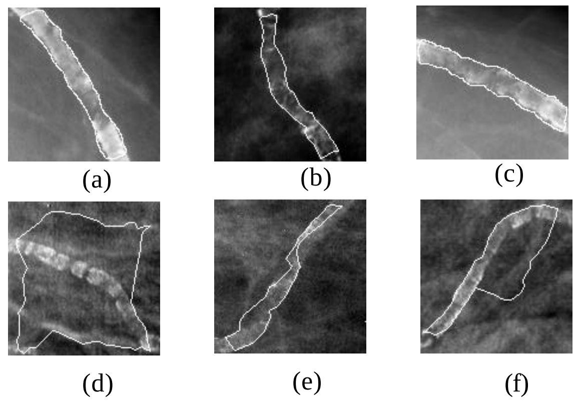


Fig. 4. Results obtained with regular snake for noisy images

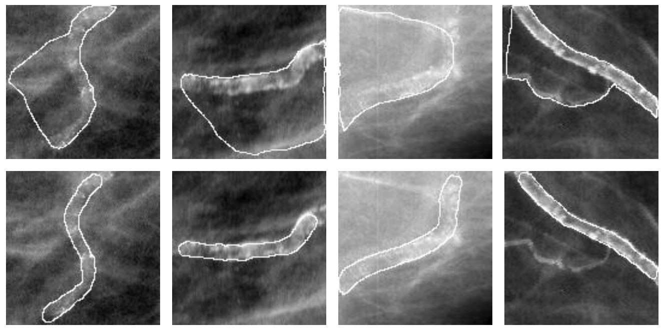
systems require a human operator to draw an initial contour close to the object. In our model, this requirement is avoided using a initial contour provided by a local deformable model, where no initial contour close to the solution is required (see example of fig 3).

Previous to geometrical deformable model application, the contour approximation (initial contour) provided by the local model is evaluated through the a posteriori probability (APP) of the contour. If the APP of the initial contour is high (above a reliable threshold,q_p) $p(x|z) \geq q_p$, then no tuning is needed (Fig 4 a-c). However, a low APP (below the reliable threshold) indicates that a tuning of the solution need to be performed through the geometrical model (Fig 4 d-f) here presented.

Iterative exploration

An easy way to minimize a global deformable model is an algorithm that moves the contour points looking for a position of lower energy value. A neighborhood of *M* x *M* of each contour point is explored and the point is moved to the location with lowest energy value. This operation is performed for every point of the contour at each iteration until the convergence criterion is done. The convergence criterion is based on the percentage of points moved in the current iteration. If this value is less than *C*, where *C* is the convergence threshold, the algorithm finishes. This algorithm can be resumed as:

```
Until Convergence criterion do
  For Pi = 1 to N do
    Explore energy value U(x)in a M x M neighborhood of
    Pi
 Move Pi to the location with minimum energy value
  End For
End Until
```



(a) (b) (c) (d)

Fig. 5. Results. Initial contour approximation with local model (upper) and tuning by geometric deformable model (bottom)

When this algorithm ends, a new contour x is obtained and the a posteriori probability $p(x \mid z)$ is evaluated. A high APP confirms that the object have been segmented correctly, and a low APP indicates that the object was not captured, presumably because it was not present in the image

Results

The new approach to a geometrical deformable contour segmentation has been validated in a database of mammograms by detecting vessels on the images. These images (called Regions of interest ROI) have a size of $L \cdot L$, begin L = 128 pixels with 1024 gray levels. A local deformable for noisy image segmentation [2] was applied to a database of 200 ROIs. Some result images are shown in Figure 4. In 24 ROIS the local deformable model was not able to segment the vessel correctly because it was captured by noise particles, as shown in Figure 4(a-c).

The threshold probability q_p for the evaluation was empirically set to 0.65 and $s_{Max} = \sqrt{2} \cdot L$, which is the diagonal of the ROI Image. The a, b and g parameters were also empirically adjusted to $a = 0.1$, $b = 0.1$ and $g = 1$.

Results of some images obtained with the geometric deformable model are presented in the Figure 5. The upper row corresponds to the initial contour (from local deformable model), and the bottom row corresponds to the tuning by the geometric model. The algorithms were implemented in C language using a Pentium II 450 Mhz, 128 Mb RAM memory and running Red Hat 6.0 linux operating system. The local deformable model applied to obtain a initial contour spent 0.20 seconds per image in average. The global deformable model spent 2 seconds in average per image to converge (for tuning the image).

Conclusions and future works

In this paper we have proposed a global deformable model that uses the geometrical information as a priori knowledge of the object. This geometrical deformable model refines the solution of a local deformable model when the local information is not enough for segmenting the object. This situation can arise due to noise perturbation or other objects in the image. Also a Bayesian framework of the model is provided in order to evaluate the contour (initial estimation and solution) through a posterior probability. A specific geometrical restriction term and a likelihood function for contour vessels in mammograms are developed. An automatic initial contour for the geometric deformable model is provided for a local deformable model, avoiding manual initialization. Finally, a minimization algorithm based on iterative exploration is proposed.

As future works we propose the following points: A faster minimization algorithm because computation of the U_{geom} term makes the convergence algorithm very slow. Moreover, an algorithm for automatic parameter adjusting could be very useful.

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