FUZZY MULTILEVEL MULTICRITERIA DECISION MAKING METHODOLOGY FOR FACILITY LAYOUT PROBLEM

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Introduction

Owing to the complexity and unstructured nature of certain layout problems, radar deployment in this case, many researchers have proposed various approaches, which have not been very successful to deal with the complexities associated with the problem. Regardless of the type of the data, there is an element of vagueness or fuzziness in it. Traditional Layout method treats these data as exact and cannot satisfy the desirability of Field Commanders in handling the real problems. We propose a multistage multi criteria method to optimize the solution. Problems like these are NP Hard and an ideal solution may not be possible. We are showing double stage methodology. Increasing the number of stages increases optimality. The multi criteria method has been compared with AHP (Analytical Hierarchy Process) and the consistency of the DM's ratings have been checked and proven using the Consistency Ratio.

Fuzzy Set and Basic Operations

The Fuzzy set theory was introduced by Zadeh (1965) to deal with problems in which absence of preciously defined criteria is involved. Formally, if X={x} is a set of objects, then the fuzzy set A on X is defined by its membership function $f_A(x)$ which assigns to each element $x \in X$, a real number in the interval [0,1] which represents the grade of membership of x in A or the degree of which x belongs to A. Thus A can be written as:

$$A = \{(f_A(x)/x) | x \in X\}; X \rightarrow [0,1].$$

A trapezoidal fuzzy number [1], as given in the above equation can be denoted by $(\alpha, \beta, \gamma, \delta)$ as shown in the fig.1. With this notation and by extension principle proposed by Zadeh (1965), the extended algebraic operations on a trapezoidal fuzzy number can be expressed as:

Its membership function $f_A(x):R\rightarrow [0,1]$ is

$$\begin{array}{ll} f_A(x) {=} (x {-} \alpha) / (\beta {-} \alpha) \text{ for } \alpha \leq x \leq \beta \\ = 1 & \text{for } \beta \leq x \leq \gamma \\ = (x {-} \delta) / (\gamma {-} \delta) & \text{for } \gamma \leq x \leq \delta \\ = 0 & \text{otherwise} \end{array}$$

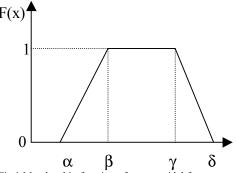


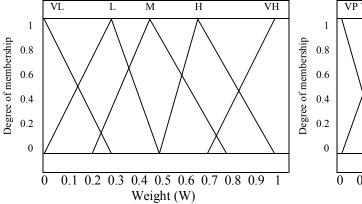
Fig.1 Membership function of a trapezoidal fuzzy no.

- Changing sign: $(\alpha, \beta, \gamma, \delta) = (-\alpha, -\beta, -\gamma, -\delta)$
- Addition: $(\alpha_1,\beta_1,\gamma_1,\delta_1) \oplus (\alpha_2,\beta_2,\gamma_2,\delta_2) =$ $(\alpha_1+\alpha_2,\beta_1+\beta_2,\gamma_1+\gamma_2,\delta_1+\delta_2)$
- Subtraction: $(\alpha_1, \beta_1, \gamma_1, \delta_1)\Theta(\alpha_2, \beta_2, \gamma_2, \delta_2)=$ $(\alpha_1-\alpha_2,\beta_1-\beta_2,\gamma_1-\gamma_2,\delta_1-\delta_2)$
- Multiplication: $(\alpha_1, \beta_1, \gamma_1, \delta_1) \otimes (\alpha_2, \beta_2, \gamma_2, \delta_2) \cong$
 - $(\alpha_1\alpha_2,\beta_1\beta_2,\gamma_1\gamma_2,\delta_1\delta_2)$
- Division: $(\alpha_1, \beta_1, \gamma_1, \delta_1) \varnothing (\alpha_2, \beta_2, \gamma_2, \delta_2) =$ $(\alpha_1/\alpha_2,\beta_1/\beta_2,\gamma_1/\gamma_2,\delta_1/\delta_2)$

The trapezoidal fuzzy members are easy to use and interpret. The concept of a linguistic variable is very useful in dealing with situations that are too complex and ill defined to be reasonably described in conventional quantitative expressions. A linguistic variable is a variable whose values are words or sentences in natural or artificial language. The approximate reasoning of fuzzy set theory can also represent the linguistic value. For example the linguistic variable of weight is {VL, L, M, H, and VH} and its membership function values are shown in fig. 2. Similarly, the membership function of the linguistic variable of ratings is {VP, VPP, P, PF, F, FG, G, GVG, and VG} are shown in fig.3. In this, paper the linguistic variables are utilized to assess the linguistic rankings given by the decision-makers (DM), as well as the linguistic weights assigned to various criteria.

Facility Selection Order

The first step to design a radar-site (facility) layout is to identify the various variables that can influence the design. Because of the vast and ill structure of FLP, it is very difficult to collect exact numerical data. Fuzzy set theory is very suitable under such situations for handling the imprecise and inexact data, yet to work in a mathematically strict and vigorous way. The experts assign numerical ratings based on a designed scale and suggest a membership function for analysis. The decision may be based on a single expert or multiple expert opinions. Depending on the expert's assessment of the linguistic variables, the selection order is made.



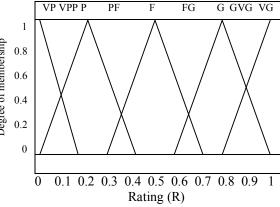


Fig.2 Membership functions of weighting set

Fig.3 Membership function of ranking set

Model Generation and Selection Criteria [2]

The concept of hierarchical structure analysis with two distinct levels is used in this paper. The following is the method we propose to solve the problem. The first level is to evaluate the fuzzy importance of the decision criteria (Technical considerations such as Slope of ground, Screening crest, Area in front of antenna etc. and Tactical considerations such as Communication, Concealment, Cover, Security, routes of approach, *etc.*)[4]. The hierarchical structure of the different factors for network layout is shown in fig.4.

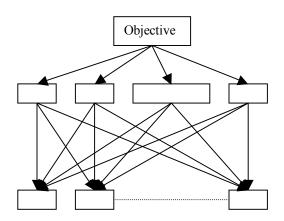


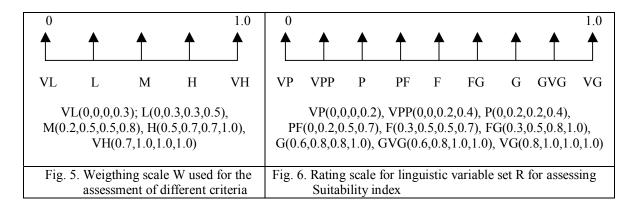
Fig. 4. Hierarchical structure with 4 criteria and sites

The second level is to assign to various sites under each decision criteria. A group of 'm' DMs is assumed to employ rating sets to evaluate preference information. The DMs assess the suitability on 'n' sites under each criterion. Let R_{ijk} be the rating assigned to site (i) by DM (j) for criteria (k). Similarly Wki be the weight given to criteria (k) by DM (j). Thus the committee has to first aggregate the ratings Riik for each site versus each criterion to form the rating Rik. Each aggregated R_{ik} for i=1,n; k=1,p; can further be weighted by a weight W_k according to the relative importance of the criteria. The fuzzy suitability index Fi of each site can be obtained by aggregating Irk and Wk for all selection criteria to form a suitability matrix. Finally, a selection routine is determined with the suitability matrix. Finally, a selection routine is determined with the suitability matrix.

Preference Rating System

The preference rating system adopted in the present problem is fuzzy members and linguistic values. The DM employs linguistic weighting set W={VL, L, M, H, VH} to evaluate the importance of the criteria through a designated rating scale in the range of 'VL' to 'VH' as shown in fig. 5. The membership

functions of the linguistic values in the weighting set W are represented graphically in fig.2. The DMs also employ a linguistic rating set R={VP, VPP, P, PF, F, FG, G, GVG, G}, as shown in fig.6, to evaluate the suitability of different sites versus various subjective criteria. The membership function of each linguistic value in the rating set is shown graphically in fig.3. In order to ensure compatibility between the fuzzy objective criteria and the linguistic ratings of the subjective criteria, the objective criteria must be converted to dimensionless indices.



The ratings of site i for objective criterion can be written as:

$$RF_i = \{F_i * [F_1^{-1} + F_2^{-1} + ... + F_n^{-1}]\}^{-1}$$
....(1)
Where F_i is the value of data flow for site i.

Thus, RF_i=R_{ip}

Aggregation of Fuzzy Assessments

The mean operator is most commonly used to aggregate the DMs fuzzy assessments.

Let

R_{ijk} =
$$(r_{ijk}^1, r_{ijk}^2, r_{ijk}^3, r_{ijk}^4)$$
, for i=1,n; j=1,m; k=1,p; be the linguistic rating assigned to site (i) by DM (j) for criteria (k) and $W_{kj} = (w_{kj}^1, w_{kj}^2, w_{kj}^3, w_{kj}^4)$, for k=1,p; j=1,m; be the linguistic weight given to subjective criteria (1, 2,..., p-1) and objective criteria (p) by DM (j).

The average linguistic rating and weight are written as:

$$R_{ik} = (1/m) * [R_{ik1} + R_{ik2} + ... + R_{ikm}]$$
 for i=1,n; k=1,p-1.

$$= (r_{ip}^{-1}, r_{ip}^{-2}, r_{ip}^{-3}, r_{ip}^{-4})$$
 for i=1,n; k=p.

$$W_k = (1/m) * [W_{k1} + W_{k2} + ... + W_{km}]$$
 for k=1,p.

$$R_{ij} = (r_{ij}^{-1}, r_{ij}^{-2}, r_{ij}^{-3}, r_{ij}^{-4})$$
 (2)

$$\begin{array}{l} R_{ik} = & (r_{ik}^{1}, r_{ik}^{2}, r_{ik}^{3}, r_{ik}^{4})......(2) \\ W_{k} = & (w_{k}^{1}, w_{k}^{2}, w_{k}^{3}, w_{k}^{4})......(3) \end{array}$$

Averaging the corresponding product over all the criteria further aggregates R_{ik} and W_k . The fuzzy suitability index of the i^{th} site can be obtained by standard arithmetic method written as:

$$F_i = (1/p) * [(R_{i1}*W_1) + (R_{i2}*W_2) + ... + (R_{ip}*W_p)].............(4)$$

It provides a trapezoidal fuzzy number $F_i = (\alpha_i, \beta_i, \gamma_i, \delta_i)$.

Ranking Value of Sites

The ranking values of the sites are determined using the maximizing set (M) and the minimizing set (N) as given below:

 $M=\{(x, f_M(x))|x \in R\}$ with the membership function values given as

$$f_M(x) = [(x-x_1)/(x_2-x_1)]^k$$
 for $x_2 \ge x \ge x_1$
=0 otherwise

N={(x, $f_N\!(x))|x\!\in\!R}$ with the membership function values given as

$$\begin{array}{ll} f_N(x) = & [(x-x_2)/(x_1-x_2)]^k & \text{for } x_2 \ge x \ge x_1 \\ = & \text{otherwise} \end{array}$$

Where
$$k>0$$
, $x_1=\inf D$, $x_2=\sup D$, $D=U_{i=1.n}D_i$, $D_i=\{x|f_{Fi}(x)>0\}$

The value of k depends on the DMs preference. The ranking value of the fuzzy suitability index can be obtained by the ranking value of the trapezoidal fuzzy number $F_i=(\alpha,\beta,\gamma,\delta)$ with the help of the equation as given below:

$$V(F_i) = [((\delta_i - x_1)/((x_2 - x_1) - (\gamma_i - \delta_i))) + 1 - ((x_2 - \alpha_i)/((x_2 - x_1) + (\beta_i - \alpha_i)))]/2....(5)$$

The ranking values of the sites can easily be calculated and a ranking value matrix can be formed to find out the facility selection order. The facility selection order can be determined with the help of the following steps:

- 1. Form a committee of DMs, then decide the facility selection criteria and identify the different sites associated with the facility layout development.
- 2. Identify the appropriate preference ratings for the importance of the facility selection criteria.
- 3. Classify the facility selection criteria into the objective and subjective categories.
- 4. Find the appropriate preference ratings for the suitability of different sites versus different criteria of facility layout.
- 5. Tabulate the weights given to the criteria by the selection makers, then get the aggregate weight W_k.
- 6. Tabulate the preference ratings assigned to the sites by the DMs, then pool them to obtain the aggregated fuzzy ratings R_{ik} of the site (i) under criteria (k).
- 7. Tabulate the flow associated with data flow associated with different sites, and then assign ratings (RF_i) to the objective criteria.
- 8. Aggregate R_{ik} and W_k with respect to each criterion, and then obtain the fuzzy suitability indices of all sites.
- 9. Calculate the ranking value $V(F_i)$ associated with each site's fuzzy suitability index F_i .
- 10. Now, send these results to another DM for further analysis.
- 11. He takes the highest ranked site, and looks at the intersection of its coverage area (another subjective criteria) with all the other sites. These criteria cannot be considered in the first level of the methodology, as it is better and convenient to resolve it in the second level, when the rankings for the first level are available.
- 12. This DM designates a new linguistic rating to all the site pairs with regard to their intersection and their ratings in the previous step.
- 13. These ratings are further aggregated to obtain final ranking values of deployment.

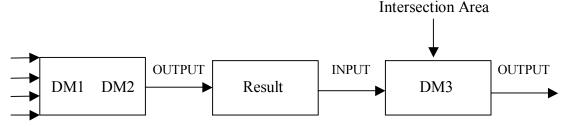


Fig. 7

Example of a radar facility layout problem

A hypothetical facility layout-planning problem [3], [5] is designed to demonstrate the procedure. The problem statement is given below.

No. of sites = 6, No. of DM's = 2 (for level I), = 1 (for level II), No. of subjective criteria = 3.

The Dm's assessment table for sites versus criteria and the weights assigned to the criteria are shown in table 1. Fuzzy suitability indices and ratings are obtained in Table 2. Level II results are calculated in table 3. The result obtained by the above methodology is 5 - 6 - 1 - 4 - 2 - 3.

Table 1

K	Communication		Conce	alment	Area in front of Radar		
DM	DM1	DM2	DM1	DM2	DM1	DM2	
W S	M	VH	Н	VH	VH	Н	
1	F	G	PF	F	G	VG	
2	G	FG	F	G	F	P	
3	P	G	G	VG	PF	F	
4	VG	G	F	F	P	G	
5	FG	F	GVG	F	PF	G	
6	G	VG	PF	G	VG	F	

Table 2

S	Communication	Concealment	Area in front of	Fuzzy Suitability Index	Ranking
			Radar		values
1	(0.45,0.65,0.65,0.85)	(0.15,0.35,0.5,0.7)	(0.7,0.9,0.9,1.0)	(0.2975,0.5165,0.5592,0.8217)	0.5056
2	(0.45,0.65,0.8,1.0)	(0.45,0.65,0.65,0.85)	(0.15,0.35,0.35,0.55)	(0.1875,0.4458,0.4833,0.7667)	0.4324
3	(0.3,0.5,0.5,0.7)	(0.7,0.9,0.9,1.0)	(0.15,0.35,0.5,0.7)	(0.215,0.4792,0.5212,0.7767)	0.4603
4	(0.7,0.9,0.9,1.0)	(0.3,0.5,0.5,0.7)	(0.3,0.5,0.5,0.7)	(0.225,0.5083,0.5083,0.7667)	0.4655
5	(0.3,0.5,0.65,0.85)	(0.45,0.65,0.75,0.85)	(0.3,0.5,0.65,0.85)	(0.195,0.4508,0.5592,0.8217)	0.4674
6	(0.7,0.9,0.9,1.0)	(0.3,0.5,0.65,0.85)	(0.55,0.75,0.75,0.85)	(0.275,0.5792,0.6092,0.8667)	0.5434

Table 3

1 able 3				
М	LEVEL I RESULT MULTIPLICATION	ANGLE OF INTERSECTION [#]	DM3 LINGUISITC RATING*	RANKING [®]
1-2	0.2186	33°	G	4
1-3	0.2327	45°	VPP	-
1-4	0.2354	22°	FG	-
1-5	0.2363	27°	VG	2
1-6	0.2747	5°	P	-
2-3	0.1990	36°	F	-
2-4	0.2013	38°	FG	-
2-5	0.2021	20°	FG	-
2-6	0.2350	25°	F	-
3-4	0.2143	35°	FG	5
3-5	0.2151	41°	P	-
3-6	0.2501	31°	G	-
4-5	0.2176	27°	FG	-
4-6	0.2530	26°	G	3
5-6	0.2540	30°	VG	1

Solution by the AHP Method

The AHP ([6], [7]) involves pair wise comparisons between various factors. The DM setting out the overall hierarchy of the decision starts the process. This structure will identify the factors to be considered as well as various alternatives in the decision. One then proceeds by carrying out pair wise comparisons, which will result in the determination of factor weights and evaluations.

Step 1: Principle of AHP relies on pair wise comparison. This comparison is carried out using a scale of 1 to 9 as follows.

- 1) Equally preferred
- 2) Equally to moderately preferred
- 3) Moderately preferred
- 4) Moderately to strongly preferred
- 5) Strongly preferred
- 6) Strongly to very strongly preferred
- 7) Very strongly preferred
- 8) Very to extremely strongly preferred
- 9) Extremely preferred

We are rating according to the jumps that are encountered in the DM ratings. For example, if a site is rated PF and it is to be compared to a site that is rated GVG, the jumps from PF to GVG are 5 $(PF \rightarrow PF \rightarrow FG \rightarrow G \rightarrow GVG)$. Since we have 7 jump levels from P to VG we are discarding 3 and 7 in the rating from 1 to 9. Therefore, 5 will be mapped on to 6.

Pair wise comparison of Concealment (by DM1 \leftrightarrow DM2)

CONCEALMENT	Site 1	Site 2	Site 3	Site 4	Site 5	Site 6
Site 1	1↔1	1/2↔1/4	1/5↔1/6	1/2↔1	1/6↔1	1↔1/4
Site 2	2↔4	1↔1	1/4↔1/4	1↔4	1/5↔4	2↔1
Site 3	5↔6	4↔4	1↔1	4↔6	1/2↔6	5↔4
Site 4	2↔1	1↔1/4	1/4↔1/6	1↔1	1/5↔1	2↔1/4
Site 5	6↔1	5↔1/4	2↔1/6	5↔1	1↔1	6↔1/4
Site 6	1↔4	1/2↔1	1/5↔1/4	1/2↔4	1/6↔4	1↔1

Step 2: The next stage is to normalize each of the columns by dividing each of the elements by the sum of the column.

CONCEALMENT	Site 1	Site 2	Site 3	Site 4	Site 5	Site 6
Site 1	.0588↔.0588	.0417↔.0370	.0513↔.0833	.0417↔.0588	.0747↔.0588	.0588↔.0370
Site 2	.1176↔.2353	.0833↔.1481	.0641↔.1250	.0833↔.2353	.0897↔.2353	.1176↔.1481
Site 3	.2941↔.3529	.3333↔.5926	.2564↔.5000	.3333↔.3529	.2242↔.3529	.2941↔.5926
Site 4	.1176↔.0588	.8330↔.0370	.0641↔.0833	.0833↔.0588	.0897↔.0588	.1176↔.0370
Site 5	.3529↔.0588	.4167↔.0370	.5128↔.0833	.4167↔.0588	.4484↔.0588	.3529↔.0370
Site 6	.0588↔.2353	.0417↔.1481	.0513↔.1250	.0417↔.2353	.0747↔.2353	.0588↔.1481

^{*}Note that the DM ratings are given considering both their intersection area as well as their previous level I rankings.

[#] Assuming 30° to be the optimum angle of intersection.

[@] Rankings are calculated by taking the highest rated pair, then the next pair that has one of the already selected sites and a new one. Repeat the algorithm to accommodate all sites.

Step 3: We can now determine the priorities of the alternative by finding the averages.

Concealment (DM1↔DM2)

Site 1: 0.0545↔0.0556 Site 2: 0.0926↔0.1879 Site 3: 0.2892↔0.4573 Site 4: 0.0926↔0.0556 Site 5: 0.4167↔0.0556 Site 6: 0.0545↔0.1879

Similarly for the other two factors Communication and Area in front of radar (DM1↔DM2)

FACTORS	Site 1	Site 2	Site 3	Site 4	Site 5	Site 6
Communication	.0599↔.1426	.1509↔.0772	.0271↔.1426	.4361↔.1426	.0898↔.0357	.1898↔.3221
Concealment	.0545↔.0556	.0926↔.1179	.2892↔.2573	.0926↔.0556	.4167↔.0556	.0545↔.1879
Area in front	.2434↔.4589	.1021↔.0151	.0575↔.0595	.0255↔.1869	.0575↔.1869	.5079↔.0690

Step 4: Now it is necessary to compare the 3 factors to determine the weightings of these 3 factors (DM1↔DM2)

FACTORS	Communication	Concealment	Area in front	
Communication	1↔1	1/4↔1	1/6↔4	
Concealment 4↔1		1↔1	1/4↔4	
Area in front $6 \leftrightarrow 1/4$		4↔1/4	1↔1	

Normalizing this matrix and averaging as in step 2&3, we obtain

Communication $0.0854 \leftrightarrow 0.4444$ Concealment $0.2435 \leftrightarrow 0.4444$ Area in front $0.6710 \leftrightarrow 0.1111$

Step 5: Now the suitability indices of the system are calculated as follows:

For Site 1 = ((.0599 * .0854 + .0545 * .2435 + .2434 * .761) + (.1426 * .4444 + .0556 * .4444 + .4589 * .1111))/2 = 0.1714

Similarly for all sites:

Sillillarly 101	all Sites.
0.1714	The Suitability Indices.
0.0918	
0.0923	
0.0929	
0.1045	
0.3022	

CONSISTENCY OF SELECTION

It is necessary to know that one has been consistent in one's pair wise comparison in order that one can accept the result of these processes, both the proposed method and the AHP. The parameter that is used to determine this is the **Consistency Ratio**. The following process can determine this factor:

Weighted Sum Vector is determined multiplying the matrix from Step 1 and one formed by the final result

1	1/2	1/5	1/2	1/6	1		0.0545	
2	1	1/4	1	1/5	2		0.0926	
5	4	1	4	1/2	5	*	0.2892	
2	1	1/4	1	1/5	2		0.0926	
6	5	2	5	1	6		0.4167	
1	1/2	1/5	1/2	1/6	1		0.0545	

6.0347	
6.0350	
6.1665	
6.0350	
6.1800	
6.0347	

The **Consistency Vector** is obtained by averaging the Weighted Sum Vector.

The **Average Value** (λ) of the elements in the consistency vector is (6.0347+6.035+6.1665+6.035+6.180+6.0347) / 6 = 6.186

Consistency Index (CI) = $(\lambda - n) / (n-1) = 0.0372$

Now the **CONSISTENCY RATIO** = CI / RI = 0.0372/1.24 = 0.03 where

n	2	3	4	5	6	7	8
RI	0.0	0.58	0.90	1.12	1.24	1.32	1.41

The CR is 0.03. This factor should be less than 0.10 if the DM's selection has been consistent. In this case one can clearly see that one has been very consistent, and therefore the evaluation is feasible. Similarly CR for DM1 and DM2 ratings for factors are 0.6 and 0.00 which are perfectly valid.

NOTE: The 2nd level can be done as in our proposed method.

Comparison of Results of Proposed method with the Traditional AHP Method

The procedure of the AHP (up to the finding of the suitability indices) for solving through reciprocal matrices is well established. Here, we see that the same results (from the suitability indices) are obtained.

However, the approach used in the paper clearly has its advantages.

- a) Better modeling of the uncertainty and the imprecision associated with the pairwise comparison process.
- b) Cognitively less demanding on the DM.
- c) Adequate reflection of the DM's attitude toward risk in their degrees of confidence in their subjective assessments due to multi leveling.

Additives

- 1. For this method the DMs are supposed to be of the same mood. Although, the difference of moods for different DMs can be analogously applied to both AHP and the method proposed by further averaging the results in Step 5.
- 2. Here we haven't considered any objective criteria. It might be considered according to the rules stated.

Conclusion

In this paper, a decision procedure is proposed to solve the FLP under Fuzzy environment. The conventional approaches are less sensitive in making effective decision. The proposed methodology considers both objective and subjective factors in such a manner that the viewpoints of total decision-

making body can be expressed without any constraints. We have also added a second level to further optimize the results. We have compared it with the AHP to further establish the solidity of the method.

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