A Semiquantitative Approach to Study Semiqualitative Systems

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Abstract: In this paper is proposed a semiquantitative methodology to study models of dynamic systems with qualitative and quantitative knowledge. This qualitative information may be composed by: operators, envelope functions, qualitative labels and qualitative continuous functions. A formalism is also described to incorporate this qualitative knowledge into these models.

We summarize the main idea of the methodology as follows: *a semiquantitative model is transformed into a set of quantitative models. Every quantitative model has a different quantitative behaviour, however, all them together may have similar qualitative behaviours*.

The methodology allows us to study all the states (transient and stationary) of a semiquantitative dynamic system. It also helps to obtain its behaviours patterns.

The completeness property is characterized by means of statistical techniques. It is also carried out a theoretical study about the reliability of the obtained conclusions.

The paper ends applying the methodology to a logistic growth model with a delay. The evolution of bacteria, mineral extraction, world population growth, epidemics, rumours, economic developments, or learning curves are real-world systems whose behaviour patterns are closely related to a logistic growth.

Keywords*:* - Dynamic systems approaches, Semiquantitative simulation, Knowledge representation, Qualitative reasoning

1. Introduction

Models of dynamic systems studied in science and engineering are normally composed of quantitative, qualitative, and semiquantitative knowledge. All this knowledge should be taken into account when these models are studied. Different levels of numeric abstraction have been proposed in the literature: purely qualitative [Kuipers (1994)], semiquantitative [Kay (1996)] [Ortega (1998)], numeric interval [Vescovi (1995)] and quantitative.

Different approximations have been proposed when the qualitative knowledge is taken into account: transformation of non-linear to piecewise linear relationships, Monte Carlo method, constraint logic programming, probability distributions, causal relations, fuzzy sets, and combination of all levels of qualitative and quantitative abstraction[Kay (1996)], [Ortega (1999)].

We are interested in the study of dynamic systems with quantitative and qualitative knowledge. The proposed methodology transforms a semiquantitative model into a family of quantitative models. A semiquantitative model may be composed of qualitative knowledge, arithmetic and relational operators, predefined functions *(log,exp,sin,...),* numbers and intervals.

A brief description of the proposed methodology is as follows: a semiquantitative model is transformed into a set of quantitative models. The simulation of every quantitative model generates a trajectory in the phase space. A database is obtained with these quantitative behaviours or trajectories. Techniques of Knowledge Discovery in Databases (KDD) are applied by means of a language to carry out queries about the qualitative properties of this time-series database. This language is also intended to classify the different qualitative behaviours of our model. This classification will help us to describe the semiquantitative behaviour of a system by means of a set of hierarchical rules obtained by means of machine learning algorithms.

The term KDD [Adriaans (1996)] is used to refer to the overall process of discovering useful knowledge from data. The problem of knowledge extraction from databases involves many steps, ranging from data manipulation and retrieval to fundamental mathematical and statistical inference, search and reasoning. Although the problem of extracting knowledge from data (or observations) is not new, automation in the context of databases opens up many new unsolved problems.

KDD has evolved, and continues to evolve, from the confluence of research in such fields as databases, machine learning, pattern recognition, artificial intelligence and reasoning with uncertainty, knowledge acquisition for expert systems, data visualization, software discovery, information retrieval, and high-performance computing. KDD software systems incorporate theories, algorithms, and methods from all of these fields.

The term *data mining* is used most by statisticians, database researchers and more recently by the business community. Data mining is a particular step in the KDD process. The additional steps in KDD process are data preparation, data selection, data cleaning, incorporation of appropriate prior knowledge and proper interpretation of the results of mining ensure the useful knowledge is derived from the data [Rastogi (1999)]. A detailed descriptions of these steps may be found in [Ortega (2000)].

The originality of our approach is that it combines in a proper way qualitative reasoning with machine learning techniques. This approach is appropriate to study all the states (transient and stationary) of a semiquantitative dynamic system. It also appropriated to obtain its behaviours patterns. However, some behaviours maybe not found with this approach, mainly, those behaviours obtained with narrowed domains of the parameters.

2 The Methodology

There has been a great deal of previous research studying the stationary state of a system, however, it is also necessary to study transient states. For example, it is very important in production industrial systems to improve their efficiency. Both states of a semiquantitative dynamic system may be studied with the proposed methodology. The methodology is shown in Figure 1.

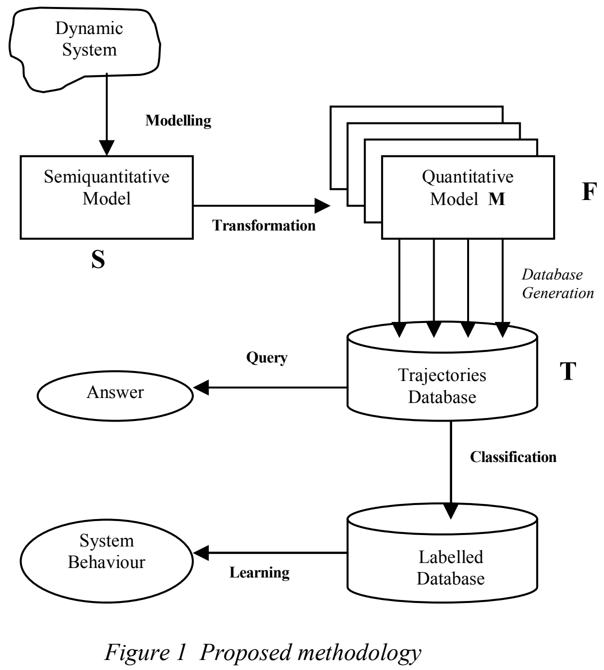

Figure 1 Proposed methodology

Starting from a dynamic system with qualitative knowledge, a semiquantitative model S is obtained.

A family of quantitative models F is obtained from S by means of the application of some transformation techniques which are bellow described.

Stochastic techniques are applied to choose a model $M \in F$. Every model M is quantitatively simulated obtaining a trajectory, which is composed by the values of all variables from its initial value until its final value, and the values of the parameters. Therefore, it contains the values of these variables in the transient and stationary states of the system.

A database of quantitative trajectories T is obtained with these quantitative behaviours. A language is proposed to carry out queries about the qualitative properties of the set of trajectories included in the database. A labelled database is obtained with the classification of these trajectories in according to a criterion.

Qualitative behaviour patterns of the system may be automatically obtained from this database by applying machine learning based on genetic algorithms. These algorithms are described in [Aguilar (1998)].

In the following sections, we are going to describe the steps of the methodology in detail. In this paper we apply the methodology to a logistic growth model with a delay. The methodology will be applied in a real computer-controlled process. It is an industrial production system. A metallurgical company is interested in modifying its steel control production system applying this methodology. They wish to improve the steel quality, and, if it is possible, to reduce the production costs. A brief description of this collaboration follows: the company has a database with measures of sensors. We are going to carry out queries for this database with the proposed language and to classify the different system behaviours.

3 Semiquantitative Models

A semiquantitative model S is represented by

$$\Phi(dx/dt,x,q,t),\ \ x(t_0) = x_0,\ \ \Phi_0(q,x_0) \tag{1}$$

being $x \in \Re^n$ the set of state variables of the system, q the parameters, t the time, dx/dt the variation of the state variables with the time, Φ constraints depending on $dx/dt,x,q,t$ and Φ_0 the set of constraints with initial conditions.

If the methodology is applied, the equations of the dynamic system (1) are transformed into a set of constraints among variables, parameters and intervals. In this paper, we are interested in those systems that may be expressed as (2) when the transformation rules are applied

$$dx/dt = f(x,p,t),\ \ x(t_0) = x_0,\ \ p \in I_p,\ \ x_0 \in I_0 \tag{2}$$

where p includes the parameters of the system and new parameters obtained by means of the transformation rules, f is a function obtained by applying the transformation rules, and I_p,I_0 are real intervals. The equation (2) is a family F of dynamic systems depending on p and x_0.

4 Qualitative Knowledge

Our attention is focused on those dynamic systems where there may be qualitative knowledge in their parameters, initial conditions and/or vector field. They constitute the semiquantitative differential equations of the system.

The representation of the qualitative knowledge is carried out by means of operators, which have associated real intervals. This representation facilitates the integration of qualitative and quantitative knowledge in a simple way, and the incorporation of knowledge from the experts [Gasca (1998)].

Qualitative knowledge may be composed of qualitative operators, qualitative labels, envelope functions and qualitative continuous functions. This qualitative knowledge and its transformation techniques are now detailed.

4.1 Qualitative Operators

These operators are used to represent qualitative parameters and initial conditions. They may be unary *U* and binary *B* operators. Every qualitative operator *op* is defined by means of an interval I_{op}, which is supplied by the experts.

Each qualitative magnitude of the system has its own unary operators. Let U_x be the unary operators for a qualitative variable *x*, i.e. $U_x=\{VN_x, MN_x, LN_x, AP0_x, LP_x, MP_x, VP_x\}$. They denote for *x* the qualitative labels *very negative, moderately negative, slightly negative, approximately zero, slightly positive, moderately positive, very positive* respectively. Let *r* be a new generated variable and let I_u be an interval defined in accordance with [Travé-Massuyès (1997)], then the transformation rule for a unary operator is as follows

$$op_u(e) \equiv \{r \in I_u\ ,\ \ e - r = 0\} \tag{3}$$

Let e_1,e_2 be two arithmetic expressions, and let op_b be a binary operator. The expression $op_b(e_1,e_2)$ denotes a qualitative relationship between e_1 and e_2. Binary qualitative operators are classified into:

- Operators related to the difference $\geq, =, \leq$. Their transformation rules are as follows:

$$\begin{aligned} e_1 = e_2 &\equiv \{\ e_1-e_2=0\ \} \\ e_1 \leq e_2 &\equiv \{\ e_1-e_2 - r=0,\ \ r \in [-\infty,0]\ \} \\ e_1 \geq e_2 &\equiv \{\ e_1-e_2 - r=0,\ \ r \in [0,+\infty]\ \} \end{aligned} \tag{4}$$

- Operators related to the quotient $\{\ll, -\!<, \sim, \approx, \gg, Vo,\ Ne, \ldots\}$. The following transformation rule is applied:

$$op_b(e_1,e_2) \equiv \{e_1-e_2*r=0\ ,\ \ r \in I_b \tag{5}$$

  where I_b is an interval defined according to [Mavrovouniotis (1988)].

In order to maintain the consistency of the model, it is necessary to add constraints to guarantee the relation among the absolute and relative order of magnitude operators. in the general case [Agell98].

is one of our are used without guaranty of consistency of both models in the general case
It is necessary to add the constraints that us in the previous transformation rules e authors should make explicit the restrictions which allow them to maintain consistency

4.2 Envelope Functions

An envelope function $y=g(x)$ (Figure 2.a) represents the family of functions included between two defined real functions, a upper one $U\!: \Re \Rightarrow \Re$ and a lower one $L\!: \Re \Rightarrow \Re$.

$$\langle L(x), U(x), I\rangle, \qquad \forall x \in I\!: L(x) \leq U(x) \tag{6}$$

where *I* is the definition domain of *g*, and *x* is the independent variable. The transformation rule applied to (6) is

$$g(x) = \alpha L(x) + (1-\alpha)\ U(x) \text{ with } \alpha \in [0,1] \tag{7}$$

where α is a new variable. If $\alpha=0 \Rightarrow g(x)=U(x)$ and if $\alpha=1 \Rightarrow g(x)=L(x)$ and any other value of α in (0,1) stands for any included value between $L(x)$ and $U(x)$.

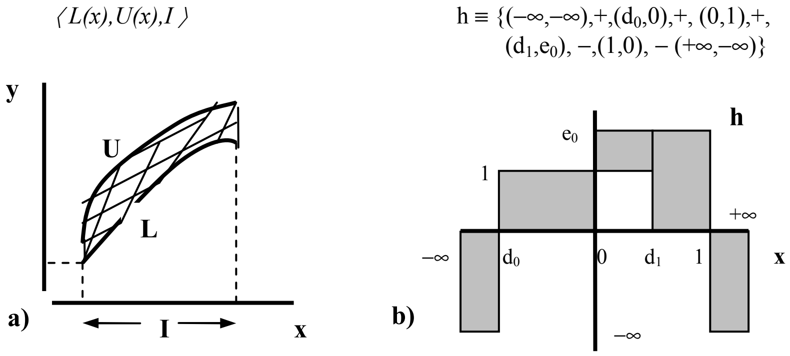

Figure 2 Qualitative functions

4.3 Qualitative Continuous Functions

A qualitative continuous function *y=h(x)* (Figure 2.b) represents a set of constraints among the values of *y* and *x* according to the properties of *h*. It is denoted by

$$y=h(x),\ \ h \equiv \{P_1,s_1,P_2,\ldots,s_{k-1},P_k\} \tag{8}$$

being P_i the points of the function. Every P_i is defined by means of (d_i,e_i) where d_i is the qualitative landmark associated to the variable x and e_i to y. These points are separated by the sign s_i of the derivative in the interval between two consecutive points. A monotonous qualitative function is a particular case of these functions where the sign is always the same $s_1=\ldots=s_{k-1}$.

The qualitative interpretation (Figure 3.a) for every $P_i=(d_i,e_i)$ of $y=h(x)$ is

$$\begin{array}{l} \text{if } x=d_i \Rightarrow y=e_i \\ \text{if } d_i < x < d_{i+1} \Rightarrow \begin{cases} s_i=+ \Rightarrow e_i < y < e_{i+1} \\ s_i=- \Rightarrow e_i > y > e_{i+1} \\ s_i=0 \Rightarrow y=e_i \end{cases} \end{array} \tag{9}$$

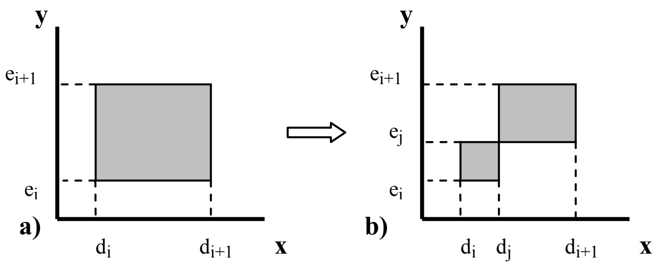

Figure 3 Qualitative interpretation of a function

The transformation rules of a qualitative continuous function are applied in three steps:

4.3.1 Normalization

The definition of the function is *completed* and *homogenised* using these continuity properties:

1) *a function that changes, its sign between two consecutive landmarks passes through a landmark whose value in the function is zero*
2) *a function whose derivative changes, its sign between two consecutive landmarks passes through a landmark whose derivative is zero*

The definition of any function (Equation 8) is always completed with: the extreme points ($-\infty$, $+\infty$), the points that denote the cut points with the axes, and where the sign of the derivative changes (a maximum or a minimum of *h*). The new landmarks d_j,e_j keep an order relationship with the old ones (Figure 3.b).

4.3.2 *Extension*

The definition of these functions is enriched by means of an automatic process, which incorporates new landmarks or qualitative labels.

This extension is carried out to diminish the uncertainty in the definition of the function, because the area of each rectangle is reduced (Figure 3.b) when a new landmark is incorporated. This reduction eliminates the total area where a quantitative function may be elected, and therefore it eliminates the uncertainty of the qualitative function.

The number of new landmarks included between each two consecutive original landmarks may be always the same. With this consideration, we don't loose the statistical representativity of the selected quantitative samples obtained for this function.

4.3.3 *Transformation*

A qualitative function is transformed into a set of quantitative functions *H*.

$$y=h(x),\ h \equiv \{P_1,s_1,P_2, \ldots s_{k-1},P_k\} \ \text{ with } P_i=(d_i,e_i) \tag{10}$$

The algorithm ***ChooseH*** is applied to obtain *H*.

ChooseH (h)
begin
  for each monotonous region in *h*
    segment=$\{P_m,\ldots,P_n\}$
    choose a value for every P_i in the *segment* verifying the constraints of *h*
  end for
end

This algorithm divides *h* into its segments. A *segment* is a sequence of consecutive points $\{P_m,\ldots,P_n\}$ separated by means of those points whose landmark $e_i=0$ or where $s_i \neq s_{i+1}$. The segments divide the function into their monotonous regions where their landmarks e_i have the same sign. The algorithm applies stochastic techniques to choose every quantitative function of *H*. These techniques are similar to the Monte Carlo method, however, the values obtained must satisfy the constraints of *h*. We use a heuristic that applies a random uniform distribution to obtain the values for every landmark of P_i.

5 Database Generation

A family *F* of quantitative models has been obtained when the transformation rules described in section 4.3 have been applied to the semiquantitative model *S*. This family depends on a set of interval parameters *p* and functions *H* defined by means of a set of quantitative points. Every particular model *M* of *F* is selected by means of stochastic techniques, and it is quantitatively simulated. This simulation generates a trajectory *r* that is stored into the database *T*.

The following algorithms are applied to obtain *T*.

ChooseModel (F)
begin
  for each interval parameter or variable of *F*
    choose a value in its interval for it
  end for

```
    for each function h of F
        H:=ChooseH(h)
    end for
end

Database generation T
begin
    T:={ }
    for i=1 to N
        M:=ChooseModel(F)
        r:= QuantitativeSimulation(M)
        T:=T ∪ r
    end for
end
```

being N the number of simulations to be carried out, and it is defined in accordance with the section *7*. Therefore, N is the number of trajectories of T.

6 Query/Classification Language

In this section, we propose a language to carry out queries to the trajectories database. It is also possible to assign qualitative labels to the trajectories with this language.

6.1 Abstract Syntax

Let T be the set of all trajectories r stored in the database. A query Q is: a quantifier operator $\forall, \exists, \aleph$ applied on T, or a basic query $[r,P]$ that evaluates *true* when the trajectory r verifies the property P.

The property P may be formulated by means of the composition of other properties using the Boolean operators $\wedge, \vee, \neg$.

$Q : \forall\, r \in T \bullet [r,P]$	$P\text{: } P_b$	$P_b\text{: } P_d$
$\mid \exists\, r \in T \bullet [r,P]$	$\mid P \wedge P$	$\mid f(L(F))$
$\mid \aleph\, r \in T \bullet [r,P]$	$\mid P \vee P$	$\mid \forall t{:}F \bullet F$
$\mid [r,p]$	$\mid \neg P$	$\mid \exists t{:}F \bullet F$

A basic property P_b may be: a predefined property P_d, a Boolean function f applied to a list L of points or intervals that verifies the formula F, or a quantifier $\forall, \exists$ applied to the values of a particular trajectory for a time t. This time may be: an instant of time, a unary time operator (i.e. a range of time), a predefined time landmark, or the list of times where the formula F is verified.

A defined property P_d is the one whose formulation is automatic. They are queries commonly used in dynamic systems. There are two predefined: *EQ* is verified when the trajectory ends up in a stable equilibrium; and *CL* when it ends up in a cycle limit.

$P_d\text{: } EQ$	$F\text{: } F_b$	$F_b\text{: } e_b$
$\mid CL$	$\mid F \,\&\, F$	$\mid e \in I$
	$\mid F \mid F$	$\mid u(e)$
	$\mid\ !F$	$\mid b(e,e)$

A formula F may be composed of other formulas combined by means of Boolean operators &,|,!.

A basic formula F_b may be: a Boolean expression e_b, or if a numeric expression e belongs to an interval, or a unary u or binary b qualitative operator.

Classification
A classification rule is formulated as a set of basic queries with labels, and possibly other expressions

$$[r,P_A] \Rightarrow A,e_{n1},\ldots \qquad [r,P_b] \Rightarrow B,e_{n2},\ldots \qquad \ldots$$

A trajectory r is classified with a label η if it verifies the property P_η.

6.2 Semantics

The semantics of every instruction of this language is translated into a query on the database. The techniques applied to carry out this transformation come from the development of compilers of language programming. A query *[r,P]* is *true* when trajectory *r* verifies the property *P*. Semantics of a query with a quantifier depends on its related quantifier. If it is $\forall$, a Boolean value *true* is returned when all the trajectories $r \in T$ verify *P*. If it is $\exists$ then *true* is returned when there is at least one trajectory $r \in T$ that verifies the property *P*. If the quantifier is $\aleph$ then returns the number of trajectories of *T* that verifies *P*.

Let $\forall t\colon F_1 \bullet F_2$ be a basic property which is *true* if during the time that F_1 is satisfied, all the values of *r* verify F_2. For $\exists$ quantifier is *true* when at least a value of *r* that satisfies F_1, also satisfied F_2. In order to evaluate a formula *F*, it is necessary to substitute its variables for their values. These values are obtained from *T*.

Let $[r,P_A] \Rightarrow A,e_{A1}$ be a classification rule. A trajectory $r \in T$ is classified with the label *A* if it verifies property P_A. The result of evaluating e_{A1} for this trajectory is also stored into the database.

7 Statistical Properties of the Conclusions

It is necessary to carry out a study about the conclusions obtained with the methodology. This must be done because the obtained answers are not complete because the methodology applies stochastic techniques. Therefore, when we claim "*there is a behaviour of the system that verifies the property P*" or "*all behaviours of the system verify the property P*", a question appears: are the obtained conclusions applicable to the real system? The answer is explored in this section.

The question to be answered is: *what is the necessary condition to secure that all the behaviours of the system verify a property P?.*

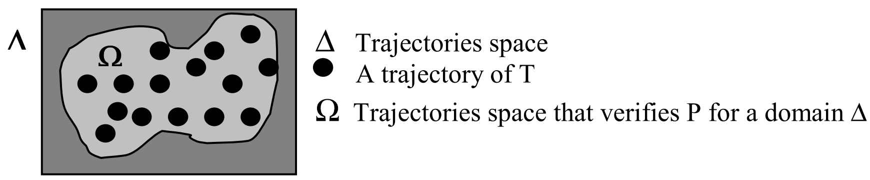

Figure 4 Trajectory space

Let Δ be the trajectory space of the system, an let Ω be the space of those trajectories of Δ that verify *P* (Figure 4). Let *Vol(s)* be the volume of a space *s*. We are interested in knowing *what is the condition that should be verified in order to secure that Vol(Δ)=Vol(Ω).*

In a schematic way, the question may be arisen as what is the condition to secure that it is true

$$\forall\, r \in T \bullet [r,P] \Rightarrow \forall\, r \in \Delta \bullet [r,P] \tag{11}$$

Statistical techniques are necessary to verify this implication.

Let *p* be the probability that a trajectory *r* verifies a property *Q* and $q=1-p$

$$p=Vol(\Omega)/Vol(\Delta) \tag{12}$$

Let *x* be a random variable. For *n* trajectories the value *x* is *n* if the $n-1$ first trajectories verifies

Q, and the *n–th* does not. Let α be the confidence degree wanted for this probability, then the expression

$$\alpha=P(x>n) \tag{13}$$

is the probability that the n first trajectories verify Q and there is a trajectory that does not verify Q among the rest of trajectories of Δ.

Theorem: *The defined probability p verifies that*

$$p \geq 1-1/(n\alpha)$$

Proof:
The expected value $E[x]$ of a random variable x is defined as follows

$$E[x]=\Sigma_{n=1}^{\infty}\ n\,p^{n-1}q = q/p\ \Sigma_{n=1}^{\infty}\ n\,p^{n} \tag{14}$$

if the geometric sum is replaced by its values, then

$$E[x]=q/p * p/(1-p)^2=1/(1-p) \tag{15}$$

On the other hand, the Chevyshev inequality follows

$$E[x]=\Sigma_{x=1}^{\infty}\ xp(x)\geq\Sigma_{x=n+1}^{\infty}\ np^{n-1} = nP(x>n)$$

if $E[x]$ is substituted by its value in (15), and if it is applied *(13)*, we will obtain

$$E[x]/n \geq P(x>n) \equiv 1/(n(1-p)) \geq P(x>n)=\alpha$$

by means of symbolic manipulation the theorem is proved

$$1/n\alpha \geq 1-p \Rightarrow p \geq 1-1/(n\alpha) \qquad \square$$

This theorem proves that: *given an confidence degree α, if we want to ensure that a property P is true for a dynamic system with a probability p, it is necessary to obtain at least n trajectories verifying it.*

Table 1 shows several examples for p and n being $\alpha=0.05$ and $\alpha=0.01$

α=0.05		α=0.01	
P=0.6	n=50	p=0.5	N=200
P=0.8	n=100	p=0.9	n=1000
P=0.98	n=1000	p=0.99	n=10000
P=0.998	n=10000	p=0.999	n=10^6

Table 1: Relation among α,p,n

A query $\exists r \in \Delta \bullet [r,P]$ may always be formulated as $\neg\ \forall r \in \Delta \bullet [r, \neg P]$, applying a property of the predicate calculus, therefore, the previous study may also be applied for this quantifier.

8 A Logistic Growth Model with a Delay

It is very common to find growth processes where an initial phase of exponential growth is followed by another phase of asymptotic approach to a saturation value (Figure 5.a). The following generic names are given: logistic, sigmoid, and s-shaped processes. This growth appears in those systems where the exponential expansion is truncated by the limitation of the resources required for this growth. They abound in the evolution of bacteria, in mineral extraction, in world population growth, in epidemics, in rumours, in economic development, the learning curves, etc.

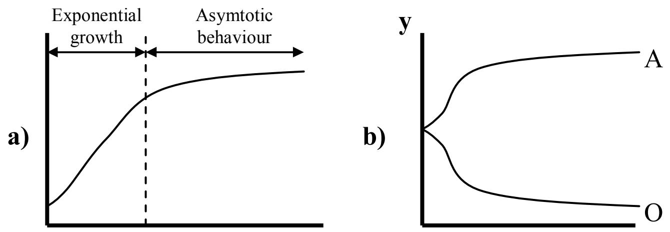

Figure 5 Logistic growth model

In the bibliography, these models have been profusely studied. There is a bimodal behaviour pattern attractor: *A* stands for normal growth, and *O* for decay (Figure 5.b).
Differential equations of the model *S* are

$$\Phi \equiv \begin{cases} dx/dt=x(n\ r-m),\ y=delay_\tau(x),\ x>0,\ r=h(y), \\ h \equiv \{(-\infty,-\infty),+,(d_0,0),+,(0,1),+, \\ (d_1,e_0),\ -,(1,0),\ -\ (+\infty,-\infty)\} \end{cases} \tag{16}$$

being *n* the increasing factor, *m* the decreasing factor, and *h* a qualitative function with a maximum point at (x_1,y_0) (Fig. 2.b). The initial conditions are

$$\Phi_0 \equiv \{\ x_0 \in [LP_x,MP_x],\ LP_x(m),LP_x(n),\ \tau \in MP_\tau,VP_\tau\}$$

where *LP,MP,VP* are qualitative unary operators for x, τ variables.
We would like to know:

1. if an equilibrium is always reached
2. if there is an equilibrium whose value is not zero
3. if all the trajectories with value zero at the equilibrium are reached without oscillations.
4. To classify the database according to the behaviours of the system.

We apply our methodology to this model. Firstly, the transformation rules to *S* are applied,

$$\Phi \equiv \begin{cases} dx/dt=x(n\ r-m),\ y=delay_\tau(x),\ x>0,\ r=H(y), \\ H,\ x_0 \in [0,3],\ \ m,n \in [0,1],\ \tau \in [0.5,10] \end{cases} \tag{17}$$

where *H* has been obtained by applying *Choose H* to *h*, and the intervals are defined in accordance with the experts' knowledge. The algorithm *Database generation T* returns the trajectories database.
The proposed queries are formulated as follows:

1. $r \in T \bullet [r,EQ]$
2. $r \in T \bullet [r,\ EQ \wedge \exists t:\ t \approx t_f \bullet\ !AP0_x(x)]$
3. $\forall r \in T \bullet [r,\ EQ \wedge \exists t:\ t \approx t_f \bullet\ AP0_x(x) \wedge \quad length(dx/dt=0)=0\ ]$
   being $AP0_x$ a unary operator of *x*. The list of points where *dx/dt=0* is the list with the maximum and minimum points. If length is *0* then there are not oscillations.

We classify the database by means of the labels:

$[r,EQ \wedge length(dx/dt=0)>0 \wedge\ \exists t : t \approx t_f \bullet\ !AP0_x(x)] \Rightarrow recovered,$

$$[r, EQ \wedge length(dx/dt=0)>0 \wedge \ \exists t : t \approx t_f \bullet AP0_x(x)] \Rightarrow retarded,$$
$$[r, EQ \wedge \exists t : t \approx t_f \bullet AP0_x(x)] \qquad \Rightarrow extinction,$$

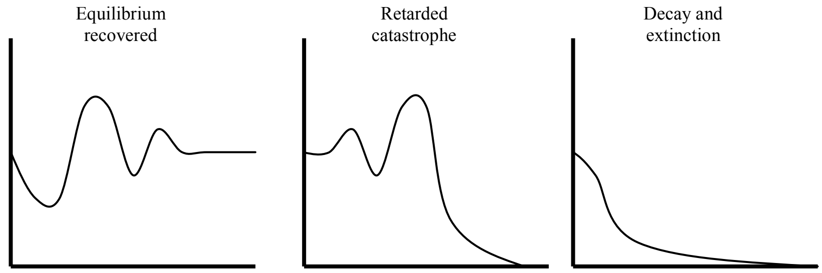

Fig. 6 Logistic growth model with a delay

They correspond to the three possible behaviour patterns of the system (Fig. 6). They are in accordance with the obtained behaviours when a mathematical reasoning is carried out [Aracil (1997)].

9 Conclusions, Potential Applications and Further Work

In this paper, a methodology is presented in order to automate the analysis of dynamic systems with qualitative and quantitative knowledge. This methodology is based on a transformation process, application of stochastic techniques, quantitative simulation, generation of trajectories database and definition of a query/classification language. There is enough bibliography that studies stationary states of dynamic systems. However, the study of transient states is also necessary. These studies are possible with the proposed language.

The simulation is carried out by means of stochastic techniques. The results are stored in a quantitative database. It may be classified by means of the proposed language. Once the database is classified, genetic algorithms may be applied to obtain conclusions about the dynamic system.

We are now studying the application of this methodology to a real computer-controlled process. It is a production industrial system of a metallurgical company interested in modifying its steel control production system applying the proposed methodology. The production engineers wish to improve the steel quality, and, if possible, reduce the production costs. The idea of this collaboration follows: the company produces steel bars of different qualities. During the whole process they add different chemical elements (Mn,Br,Zn,…) to improve the quality of the bar. This adding process it is carried out according to the experts experience but it is not automated. They measure with sensors for every bar different values since the scrap is introduce into the blast furnace until the bar is obtained. These values are stored into a database. We are going to carry out queries for this database with the proposed language and to classify the different system behaviours, and if it is possible to obtain a model of the system. This collaboration is now developing and in forthcoming papers, we will describe this system in detail and the conclusions we shall obtain.

In the future, we are going to enrich the query/classification language with: operators for comparing trajectories among them, temporal logic among several times of a trajectory, more type of equations, etc.

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