Generalized modifiers as an interval scale: towards adaptive colorimetric alterations

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Abstract. In this article, new tools to represent the different states of a same knowledge are described. These states are usually expressed through linguistic modifiers that have been studied in a fuzzy framework, but also in a symbolic context.
The tools we introduce are called *generalized symbolic modifiers*: they allow linguistic modifications. A first beginning of this work on modifiers has been done by Akdag & al and this paper is the continuation. Our tools are convenient and simple to use; they assume interesting mathematical properties as order relations or infinite modifications and, moreover, they can be seen as an interval scale. Besides, they are used in practice through a colorimetric application and give very good results. They act as a link between modifications expressed with words and colorimetric alterations.

Keywords. symbolic framework, modifiers, interval scales.

This paper should be considered in the **Paper Track**.

Conference topics: AI Foundations and Knowledge Representation; Reasoning Models: Reasoning under Uncertainty; Natural Language Processing.

1 Introduction

When imperfect knowledge has to be expressed, modifiers are often used to translate the many states of a same knowledge. For example, we can associate the modifiers "very", "more or less", "a little", etc. with the knowledge "young". These intermediate descriptions have been called by Zadeh [11] *linguistic hedges* or *linguistic modifiers* and have been taken up by Eshragh & al [6] and Bouchon–Meunier [4] notably.

It seems to be interesting to define modifiers that would allow to modify values at will for a given application. In this paper, new tools, the generalized symbolic modifiers, are introduced for this kind of modification. These tools have very interesting mathematical properties and are also fully appropriate in practice.

The paper is organized as follows: section 2 is devoted to the different existing approaches about modifiers. In particular, we briefly present modifiers defined in a fuzzy framework [4], [5], [11] but also in a symbolic framework [1]. Our propositions about generalized symbolic modifiers are described in section 3. In particular, we assure that very few conditions are necessary to use and apply them and we explain how they can be considered as an interval scale. In section 4 the application developed is detailed. Indeed, generalized modifiers are very useful in colorimetry and allow to propose adaptive colorimetric alterations. Finally, section 5 concludes this study.

2 Modifiers and measure scales

There are especially two kinds of approaches about linguistic modifiers: fuzzy and symbolic approaches. The first ones deal with fuzzy logic and represent the modifiers as modifications of membership functions while the others represent them as modifications of values on a scale basis.

2.1 Context in fuzzy logic

Zadeh has been one of the pioneers in this domain [12]. He has proposed to model concepts like "tall" or "more or less high",... with fuzzy subsets and, more precisely, with their representation i.e. membership functions. Afterwards, some authors like Desprès have proposed an approach aiming at a classification of the fuzzy modifiers [5]. Desprès has defined classes or families of modifiers: the intensive ones and the extensive ones. The family of intensive modifiers reinforces the initial value while the other family weakens it. The author distinguishes also a third kind of modifiers composed of two sub-families: the concentrating and the contrasting ones. In the last-mentioned case, the membership functions overlap with each other and the associated modifiers are considered as extensive ones. The figure 1 sums these different cases up.

Besides, the modifiers can be studied in a symbolic framework. Let us now have a look at what has been done concerning more symbolic approaches.

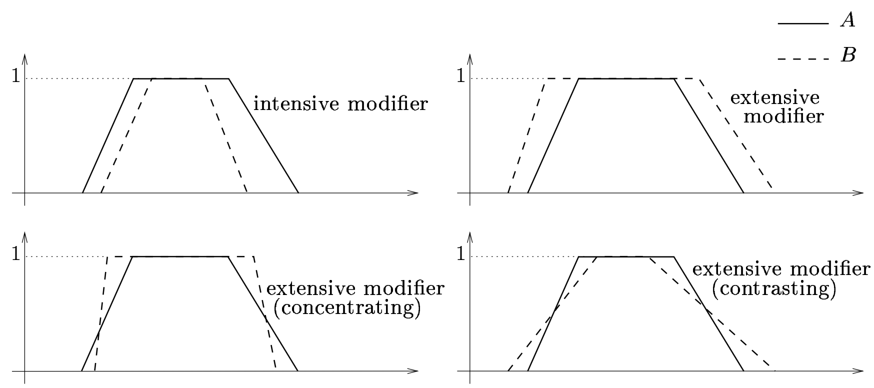

Fig. 1. Examples of modifiers enabling us to go from A to B.

2.2 Symbolic context

Akdag & al suggest to model modifiers in a symbolic context [1], [2]. Indeed, adverbs evaluating the truth of a proposition are often represented on a scale of linguistic degrees. They propose tools, i.e. symbolic linguistic modifiers, to combine and aggregate such symbolic degrees. The authors also introduce the notion of intensity rate associated to a linguistic degree on a scale basis.

Formally, Akdag & al define a symbolic linguistic modifier m as a semantic triplet of parameters. Let a be a symbolic degree and b a scale basis[1]; to a pair (a, b) corresponds a new pair (a', b') obtained by linear transformation depending on m. a' is the modified degree and b' the modified scale basis:

$$(a', b') = f_{m(\text{quantifier},\text{nature},\text{mode})}(a, b)$$

The quantifier (called λ) expresses the strength of the modifier, the nature is the way to modify the scale basis (i.e. dilation, erosion or conservation) and the mode is the sense of modification (weakening or reinforcing). Besides, they associate to each linguistic degree D of range a on a scale b its intensity rate, the proportion $\text{Prop}(D) = a/(b-1)$.

The authors establish (as Desprès does in fuzzy logic) a classification of their modifiers. They define three families:

- weakening modifiers: they give a new characterisation which is less strong than the original one. They are divided into two subfamilies: the ones that erode the basis — the eroding ones (called $\text{EG}(\lambda)$ and $\text{EG}(\lambda^{\frac{1}{2}})$) — and the ones that dilate the basis — the dilating ones (called $\text{IG}(\lambda)$ and $\text{IG}(\lambda^{\frac{1}{2}})$),
- reinforcing modifiers: they give a new characterisation which is stronger than the original one. They are divided into two subfamilies: the ones that erode

[1] The authors consider that a degree's scale denoted b represent a finite number of ordered degrees $0, 1, 2, \ldots b-1$. $b-1$ is thus the biggest degree of the scale.

the basis (called ED(λ) and ED($\lambda^{\frac{1}{2}}$)) and the ones that dilate the basis (called ID(λ) and ID($\lambda^{\frac{1}{2}}$)),

– central modifiers: they give a new characterisation which is similar to the original one, but more or less precisely. We won't deal with these ones as we are not interested in them in this paper.

The two first families are gathered and briefly defined in figure 2 for a best understanding.

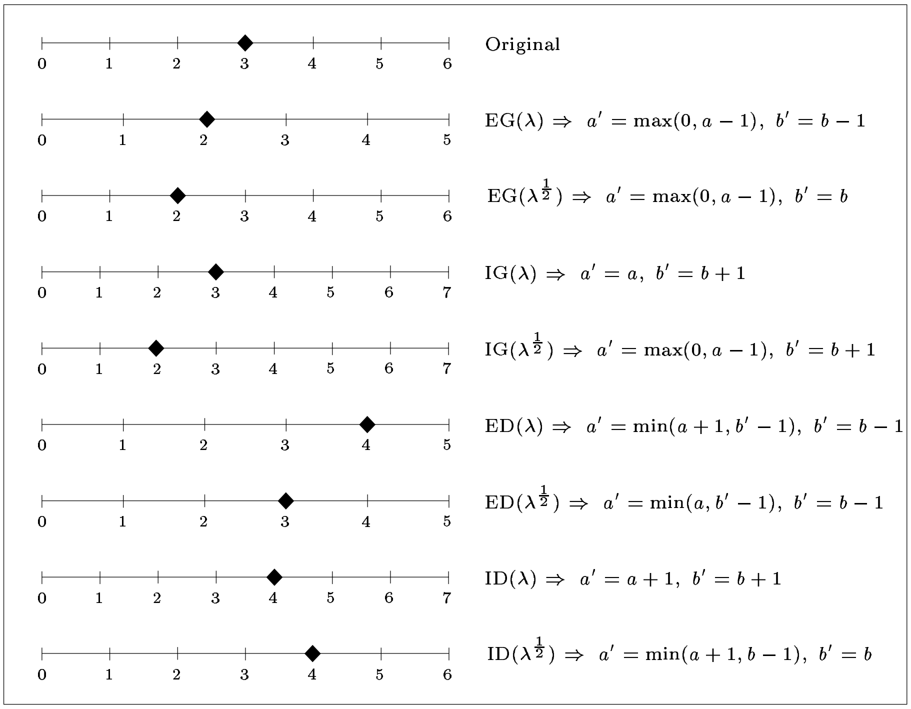

Fig. 2. Summary and comparison of the symbolic linguistic modifiers.

2.3 Measure scales

As we have seen, the symbolic modifiers just introduced above act on scales. That is why it seems now appropriate to look into the existing works about this subject.

Measure scales are often used by statisticians [3] but also by researchers in fuzzy logic, like Grabisch [7]. [3] defines four measure scales that are described below:

The nominal scale. The codes that identify the variable are independent from each other. There is no order relation. Examples: sex (F/M); marital status (single/married/widower), etc.

The ordinal scale. The codes allow us to establish an order relation between them. Examples: groups of age (under 18, between 18 to 30, 30 to 50, etc.); education level (primary, secondary, academic...).

The interval scale. The conditions are the same as in the previous case, but, moreover, the codes must be uniformly distributed on the scale, i.e. the intervals between codes have an importance, a meaning. Example: ambient temperature in °C.

The ratio scale. The conditions are the same as in the previous case, but, moreover, the position of zero is important, i.e zero is an absolute zero. Examples: company turnover; programme execution time, etc.

Let us see now our propositions about symbolic modifiers and how they can be considered as measure scales.

3 Generalized symbolic modifiers

Here we propose a generalization of symbolic linguistic modifiers presented in section 2.2. We clarify the role of the quantifier λ and we propose more general families of modifiers, for any given quantifier. Moreover, we establish a link between our modifiers and one of the measure scales studied above.

3.1 Definition

We associate to the notion of modifier a semantic triplet {radius, nature, mode}. The radius ρ ($\rho \in \mathbb{N}^*$) represents the strength of the modifier. As defined in section 2.2, the nature is the way to modify the scale basis (i.e. dilation, erosion or conservation) and the mode is the sense of modification (weakening or reinforcing). The more ρ increases, the more powerful the modifier.

The triplet {radius, nature, mode}, i.e. (ρ, n, o) is sufficient to identify the modifier uniquely, but in a more general way, we define a generalized symbolic modifier as a function allowing us to go from a pair (a, b) to a new pair (a', b').

Definition 1 *Let (a, b) be a pair belonging to $\mathbb{N} \times \mathbb{N}^*$. A generalized symbolic modifier is defined as:*

$$\begin{array}{rcl} \mathbb{N} \times \mathbb{N}^* & \rightarrow & \mathbb{N} \times \mathbb{N}^* \\ (a, b) & \mapsto & (a', b') \quad \textit{with} \quad a < b \quad \textit{and} \quad a' < b' \end{array}$$

a is a degree on a uniformely distributed scale b.

The definitions of our modifiers are summarized in the table 1.

Table 1. Summary of reinforcing and weakening generalized modifiers.

MODE / NATURE	Weakening	Reinforcing
Erosion	$a' = \max(0, a-\rho)$ $b' = \max(1, b-\rho)$ $\boxed{\mathbf{EW}(\rho)}$	$a' = a$ $b' = \max(a+1, b-\rho)$ $\boxed{\mathbf{ER}(\rho)}$ $a' = \min(a+\rho, b-\rho-1)$ $b' = \max(1, b-\rho)$ $\boxed{\mathbf{ER'}(\rho)}$
Dilation	$a' = a$ $b' = b+\rho$ $\boxed{\mathbf{DW}(\rho)}$ $a' = \max(0, a-\rho)$ $b' = b+\rho$ $\boxed{\mathbf{DW'}(\rho)}$	$a' = a+\rho$ $b' = b+\rho$ $\boxed{\mathbf{DR}(\rho)}$
Conservation	$a' = \max(0, a-\rho)$ $b' = b$ $\boxed{\mathbf{CW}(\rho)}$	$a' = \min(a+\rho, b-1)$ $b' = b$ $\boxed{\mathbf{CR}(\rho)}$

3.2 Order relation

The generalized modifiers assume a partial order relation $\trianglelefteq$. First we define what exactly this relation is.

Preliminary notation: If we consider a modifier m, a the original degree on a scale b and a' the modified degree on the modified scale b' then we denote:

$$a' = m(a) \text{ and } b' = m(b)$$

Definition 2 *Let m_1 and m_2 identified by (ρ_1, n_1, o_1) and (ρ_2, n_2, o_2) be two modifiers. Prop$(m_1) = \dfrac{m_1(a)}{m_1(b)-1}$ is comparable with Prop$(m_2) = \dfrac{m_2(a)}{m_2(b)-1}$ if and only if*

$$\begin{cases} \textit{Prop}(m_1) \leq \textit{Prop}(m_2) \textit{ for any given } a \textit{ and } b \\ \textit{or} \\ \textit{Prop}(m_2) \leq \textit{Prop}(m_1) \textit{ for any given } a \textit{ and } b \end{cases}$$

Definition 3 *Two modifiers m_1 and m_2 identified by (ρ_1, n_1, o_1) and (ρ_2, n_2, o_2) entail an order relation if and only if Prop$(m_1) = \dfrac{m_1(a)}{m_1(b)-1}$ is comparable with Prop$(m_2) = \dfrac{m_2(a)}{m_2(b)-1}$, for any given a and b. Formally, the relation $\trianglelefteq$ is defined as follows:*

$$m_1 \trianglelefteq m_2 \Leftrightarrow \frac{m_1(a)}{m_1(b)-1} \leq \frac{m_2(a)}{m_2(b)-1} \quad \textit{for any given } a \textit{ and } b$$

Let us note that if a pair of modifiers (m_1, m_2) are in relation to each other, the comparison between their intensity rates (Prop(m_1) and Prop(m_2)) is possible obviously because these intensity rates are rational numbers but particularly because the unit is the same, for all a and b. Indeed, the degrees are uniformly distributed on the scales.

Furthermore, the binary relation $\trianglelefteq$ over the generalized modifiers is a partial order relation: it is easy to see that $\trianglelefteq$ is reflexive, transitive and antisymmetric, as the order relation $\leq$.

If we compare the generalized modifiers in pairs, we establish a partial order relation between them that we express as usual in fuzzy logic through a lattice. The relation is only partial because some modifiers can not be compared with some others. The figure 3 shows the lattice.

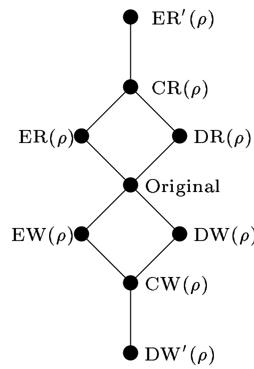

Fig. 3. Lattice for the relation $\trianglelefteq$.

3.3 Finite and infinite modifiers

Moreover, we can notice that some modifiers can modify the initial value towards infinity. That is the case of two modifiers: DW(ρ) and DR(ρ).

Definition 4 *We define an infinite modifier m identified by (ρ, n, o) as follows:*

$$m \textit{ is an infinite modifier } \Leftrightarrow \begin{cases} (\forall \rho \in \mathbb{N}^*,\ \mathit{Prop}(\rho+1, n, o) > \mathit{Prop}(\rho, n, o)) \\ \mathit{or} \\ (\forall \rho \in \mathbb{N}^*,\ \mathit{Prop}(\rho+1, n, o) < \mathit{Prop}(\rho, n, o)) \end{cases}$$

This means that the modifier will always have an effect on the initial value.

Definition 5 *We define a finite modifier m identified by (ρ, n, o) as follows:*

$$m \textit{ is a finite modifier } \Leftrightarrow \begin{cases} \exists \rho \in \mathbb{N}^* \textit{ such as } \forall \rho' \in \mathbb{N}^* \textit{ with } \rho' > \rho \\ \qquad \mathit{Prop}(\rho', n, o) = \mathit{Prop}(\rho, n, o) \end{cases}$$

This means that, starting from a certain rank, the modifier has no effect on the initial value.

Depending on what we want to do, finite or infinite modifiers can be very useful.

3.4 The modifiers as an interval scale

Our modifiers correspond to one measure scale: the interval one. Indeed, we work on scales with an order relation (the degrees are ordered) and, as it is said in the definition, there is a condition about the degrees' distribution. They are uniformly distributed on the scale.

4 Application

An interesting implementation of our generalized modifiers lies in colorimetrics. We propose a piece of software that is dedicated to colour modification according to colorimetric qualifiers (like "dark", "bright", "bluish"...) and linguistic terms for the modification (as "much more", "a little bit less"...). For example, the user can ask for a red "a little bit more bluish".

4.1 Context

To modify a colour, we modify its colorimetric components expressed in a certain space (either RGB-space for Red-Green-Blue, or HLS-space for Hue-Lightness-Saturation...). The space we have chosen is HLS for many reasons explained in [9]. We increase or decrease the components thanks to our generalized modifiers. We establish a link between the linguistic terms and the symbolic generalized modifiers.

A colour is associated to three symbolic scales, one scale for each colorimetric component. For example, to display on the user's screen a "brighter" green, we can use the modifier CR(ρ) with ρ equal a certain value (depending on the total number of linguistic terms) that increases the value of L (Lightness). The three components H, L and S can be modified at the same time or not, depending on the selected qualifier. Indeed, some qualifiers (like "gloomy", for example) require a modification of only one component, while some others (like "bluish", for example) require modifications of more than one component.

To simplify the modification, we split the range of a component (i.e. [0,255]) into three equal parts, and we split the parts into a certain number of sub-parts — this number depending on the quantity of linguistic terms we have, since each sub-part is associated to a linguistic term. The figure 4 shows a very simple example of qualifiers associated to the component L.

A deeper study of this process is explained in [10] and a comparison between this symbolic approach with a fuzzy one is done in [8].

4.2 Which modifiers for what modification?

We can use all kinds of our generalized modifiers in this application. But, it is preferable not to use the infinite ones because the very principle of these modifiers — for example DR(ρ) that reinforces — is never to reach the top of the scale, i.e $b-1$. It means that, knowing that a component takes its values

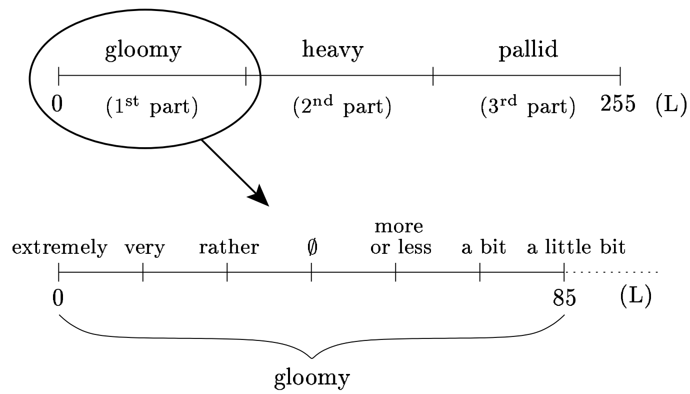

Fig. 4. Qualifiers associated to parts of L space.

between 0 and 255, 255 will never be reached. In fact, with the approximation of the calculus, it will probably be reached, but, theoretically, we don't want to use these modifiers since we do want to reach the maximum.

So, the modifiers we use are the finite ones and, depending on where the initial value of the component is, we use the most powerful modifiers or the less powerful ones.

– In the cases where the initial value of the colorimetric component has to be set to the first or the third part (for example if the user has asked for a *gloomier* or a *more pallid* colour), we use the most powerful modifiers, i.e. ER'(ρ) and/or CR(ρ) and DW'(ρ) and/or CW(ρ). The figure 5 shows this case.
– In the cases where the initial value of the colorimetric component has to be set to the second part (for example if the user has asked for a *heavier* colour), we use the less powerful modifiers, i.e. ER(ρ) and EW(ρ). The figure 6 shows this case.

The reason of these choices is simple: the biggest distance between an initial value and the "2$^{\text{nd}}$ part" is twice as short as the biggest distance between an initial value and the "1$^{\text{st}}$ part" (or the "3$^{\text{rd}}$ part"). So, a slow approach when the initial value is close to the value to be reached has been favored. It is a way to compensate the differences between distances.

4.3 Learning for an adaptive alteration

As the perception of colours is very subjective, we have added to our software a learning process. Indeed, a "very much more pallid red" for one person can be interpreted as a "little bit more pallid red" or even as a "much heavier red" for another person. That is why it is possible to change the association between

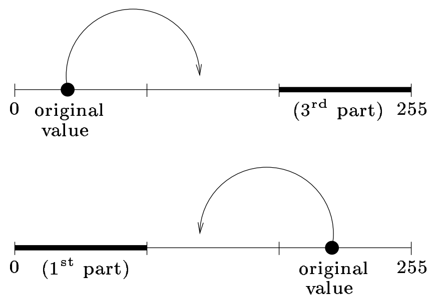

Fig. 5. Case of "big jumps" performed by the most powerful generalized modifiers.

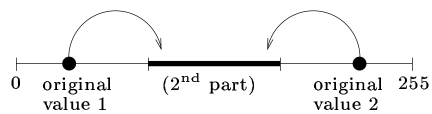

Fig. 6. Case of "small jumps" performed by the less powerful generalized modifiers.

the colours modifications (through the generalized symbolic modifiers) and the linguistic terms. In our application, a linguistic term corresponds to both a qualifier ("pallid", ...) and a linguistic quantifier ("much more", ...). A particular association will reflect the perception of a particular user. This process is carried out thanks to an internal representation of the modifications through a graph. More explanations about that will be given in a further work.

Besides this learning process, one interesting thing is that this process of modification can be done in both directions. Indeed, the user can ask for a certain colour through a linguistic expression and he obtains it, but, on the contrary, from a colour, the software can give a linguistic expression composed of a qualifier and a modifier.

Furthermore, this application could be inserted in another one that would classify pictures by predominant colour, for example.

5 Conclusion

We have presented new symbolic modifiers: the *generalized symbolic modifiers*, coming from the linguistic symbolic modifiers introduced by Akdag & al [1]. We have seen that they embody some good mathematical properties and we notably use them in a colorimetric application.

Moreover, we believe that the modification process can be seen as an aggregation process. Indeed, for us, to modify is equivalent to aggregating an initial value with an expression of the modification. In the colours application, this can be summed up as shown on the figure 7.

The symbol "+" on the figure symbolizes the aggregation process.

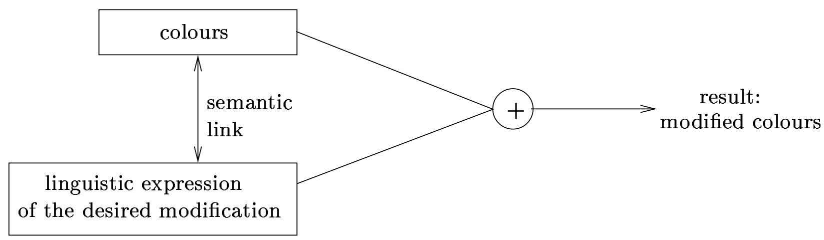

Fig. 7. Link between aggregation and modification.

The modifiers which have been introduced in this paper can help a lot in an aggregation process. An interesting perspective would be to pursue our research in this domain and imagine an aggregator defined as a composition (in the mathematical sense) of our modifiers.

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